

# Radiatively Driven Stellar Winds from Hot Stars

A stellar wind is the continuous, supersonic outflow of matter from a star. Among the most massive stars – which tend also to be the hottest and most luminous – the winds can be very strong, with important consequences for both the star’s own evolution, as well as for the surrounding interstellar medium. Such hot-star winds are understood to be driven by the pressure of the star’s emitted radiation.

Our sun has a solar wind, but it is so tenuous and transparent that it would be difficult to detect directly from a more distant star. However, many stars have winds that are dense enough to be opaque at certain wavelengths of the star’s radiation, and this makes it possible to study them remotely through careful interpretation of the observed stellar spectra. The winds from massive, hot stars – with surface temperatures above about 10,000 K – form a well-defined class, distinct from the winds from cooler stars (which are reviewed in a separate article), and characterized by the central role of the star’s own radiation in driving the mass outflow. The high temperature of such stars means they have a very high surface brightness. Because light carries momentum as well as energy, this high surface brightness imparts a force to the atoms that scatter the light. At the level of the star’s atmosphere where this force exceeds the inward force of the stellar gravity, material is accelerated upward and becomes the stellar wind.

An important aspect of this radiative driving process is that it stems mostly from *line* scattering, wherein an electron is shuffled between two discrete, bound energy levels of an atom. In a static medium such scattering is confined to radiation with a photon energy near the energy difference between the levels, corresponding to a range of wavelengths near a distinct, line-center value. But in an accelerating stellar wind flow, the Doppler effect shifts the resonance to increasingly longer wavelengths, allowing the line scattering to sweep gradually through a much broader portion of the stellar spectrum. This gives the dynamics of such winds an intricate feedback character, in which the radiative driving force that accelerates the stellar wind depends itself on that acceleration.

## OVERVIEW AND COMPARISON WITH THE SOLAR WIND

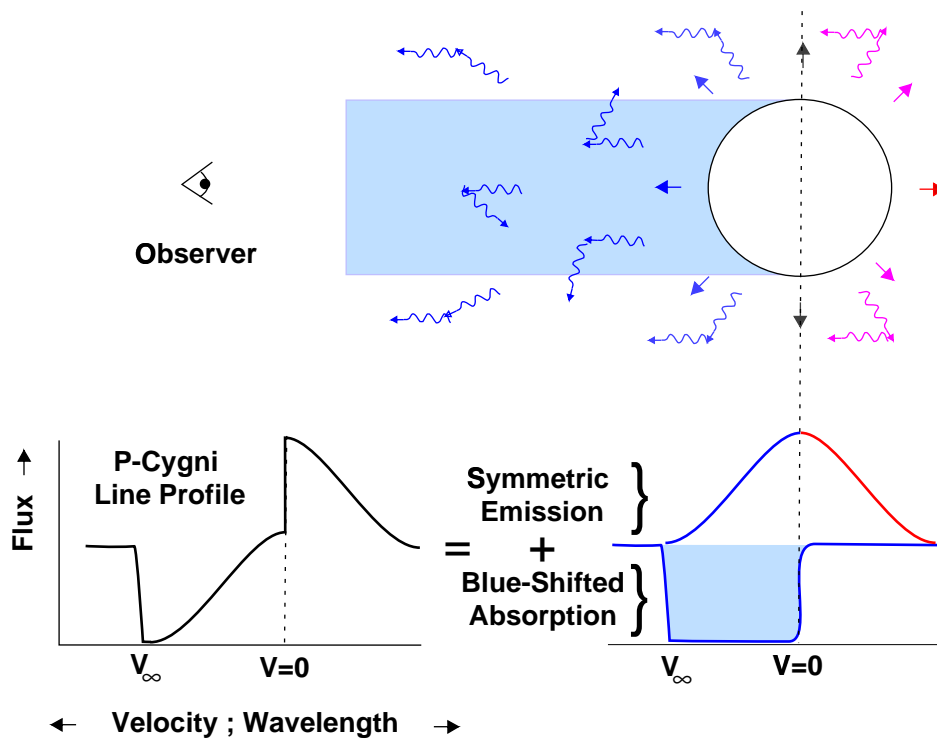
The general concept that stars can sometimes lose or eject matter is apparent from some spectacular, catastrophic examples, like nova or supernova explosions. But the specific notion of a *continuous*, quiescent-phase stellar wind outflow stems largely from discovery, during the 1950's and 60's, of the *solar* wind. The sun is a relatively low-mass, cool star with a surface temperature about 6000 K; but curiously its wind arises from pressure-expansion of the very hot, million-degree solar corona, which is somehow superheated by the mechanical energy generated from convection in the sun's subsurface layers. By contrast, high-mass stars with much hotter surface temperatures (10,000-100,000 K) are thought to lack the strong convection zone needed to heat a circumstellar corona. Their stellar winds thus remain at temperatures comparable to the star's surface, and so lack the very high gas-pressure needed to drive an outward expansion against the stellar gravity. However such hot stars have a quite high radiative flux, since by the Stefan-Boltzmann law this scales as the fourth power of the surface temperature. It is the pressure of this *radiation* (not of the gas itself) that drives the wind expansion.

The typical flow speeds of hot-star winds – up to about 3000 km/s – are a factor few faster than the 400-700 km/s speed of the solar wind. But the inferred mass loss rates of hot stars greatly exceed – by up to a factor of a billion! – that of the sun. At the sun's current rate of mass loss, about  $10^{-14} M_{\odot}/yr$ , its mass would be reduced by only  $\sim 0.01\%$  during its entire characteristic lifespan of 10 billion ( $10^{10}$ ) years. By contrast, even during the comparatively much shorter, few million ( $10^6$ ) year lifetime typical for a massive star, its wind mass loss at a rate of up to  $10^{-5} M_{\odot}/yr$  can substantially reduce, by a factor of two or more, the original stellar mass of a few times  $10 M_{\odot}$ . Indeed, massive stars typically end up as “Wolf-Rayet” stars, which often appear to have completely lost their original envelope of hydrogen, leaving exposed at their surface the elements like carbon, nitrogen, and oxygen that were synthesized by nuclear processes in the stellar core. (Much of this cumulative mass loss might also occur during relatively brief “superwinds”, or episodic ejections as a “Luminous Blue Variable”; see the article on episodic stellar mass loss.) In addition to directly affecting the star's own evolution, hot-star winds often form “wind-blown bubbles” in nearby interstellar gas. Overall they represent a substantial contribution to the energy, momentum, and chemical enrichment of the interstellar medium in the Milky Way and other galaxies.

## WIND DETECTION THROUGH ASYMMETRIC, P-CYGNI LINE PROFILES

The high density of hot-star winds means that the expanding material is itself opaque, or *optically thick*, to scattering in many atomic spectral-line transitions. When observed as part of a stellar spectrum, radiation that is line-scattered within the wind develops a characteristic line profile, known as a “P-Cygni profile”, after the star P-Cygni in which its significance as a signature of mass outflow was first broadly recognized.

Figure 1 illustrates the formation of a P-Cygni line-profile. Wind material approaching an observer within a column in front of the star has its line-resonance blue-shifted by the Doppler effect. Thus scattering of stellar radiation out of this direction causes an absorption trough, or reduction in the observed flux, on the blue side of the line profile. However, from the lobes on either side of this absorption column, wind material can also scatter radiation *toward* the observer. Since this can occur from either



Schematic illustration of the formation of a P-Cygni type line-profile in an expanding wind outflow.

the approaching or receding hemisphere, this scattered radiation can either be blue-shifted or red-shifted. The associated extra flux seen by the observer thus occurs as a symmetric emission component on both sides of the line center. Combined with the reduced blue-side flux, the overall profile has a distinctly asymmetric form, with apparent net blueward absorption and redward emission. Moreover, the wavelength of the blue edge of the absorption provides a quite unambiguous measure of the asymptotic wind speed,  $v_\infty$ .

## THE RADIATIVE FORCE FROM ELECTRON SCATTERING

A radiative force results from the material interception of radiative momentum. A particularly simple case is scattering by free electrons, which is a “gray”, or frequency-independent, process. Since gray scattering cannot alter the star’s total luminosity  $L$ , the radiative energy flux at any radius  $r$  is simply given by  $L/4\pi r^2$ . This corresponds to a radiative *momentum* flux of  $L/4\pi r^2 c$ , where  $c$  is the speed of light. The material interception of this flux is characterized by an interaction cross section per unit mass,  $\kappa$ , which is formally named the “mass-absorption-coefficient”, but more succinctly is also commonly called the “opacity”. For electron scattering in an ionized medium, the opacity is simply a constant given by  $\kappa_e = \sigma_e/\mu_e$ , where  $\sigma_e$  ( $= 0.66 \times 10^{-24} \text{ cm}^2$ ) is the classical Thompson cross-section, and the mean atomic mass per free electron is  $\mu_e = 2m_H/(1 + X)$ , with  $m_H$  and  $X$  the mass and mass-fraction of hydrogen. This works out to a value  $\kappa_e = 0.2(1 + X) = 0.34 \text{ cm}^2/\text{g}$ , where the latter result applies for the standard (solar) hydrogen mass fraction  $X = 0.72$ . The product of this opacity and the radiative momentum flux yields the radiative acceleration (force-per-unit-mass)

from free-electron scattering,

$$g_e(r) = \frac{\kappa_e L}{4\pi r^2 c}.$$

It is of interest to compare this with the star's gravitational acceleration, given by  $GM/r^2$ , where  $G$  is the gravitation constant, and the  $M$  is the stellar mass. Since both accelerations have the same  $1/r^2$  dependence on radius, their ratio is spatially constant, fixed by the ratio of luminosity to mass,

$$\Gamma_e = \frac{\kappa_e L}{4\pi GMc}.$$

This ratio, sometimes called the Eddington parameter, thus has a characteristic value for each star. For the sun it is very small, of order  $2 \times 10^{-5}$ , but for hot, massive stars it is often within a factor of two below unity. As noted by Eddington, electron scattering thus represents a basal radiative acceleration that effectively counteracts the stellar gravity. The limit  $\Gamma_e \rightarrow 1$  is known as the Eddington limit, for which the star would become gravitationally unbound.

It is certainly significant that hot stars with strong stellar winds have  $\Gamma_e$  only a factor two or so below this limit, since it suggests that only a modest additional opacity could succeed in fully overcoming gravity to drive an outflow. But it is important to realize that a stellar wind represents the outer envelope outflow from a nearly static, gravitationally bound base, and as such is not consistent with an *entire* star exceeding the Eddington limit. Rather the key requirement for a wind is that the driving force increase naturally from being smaller to larger than gravity at some radius near the stellar surface. We shall now see that the force from *line*-scattering is ideally suited for just such a spatial modulation.

## THE DOPPLER-SHIFTED RESONANCE OF LINE-SCATTERING

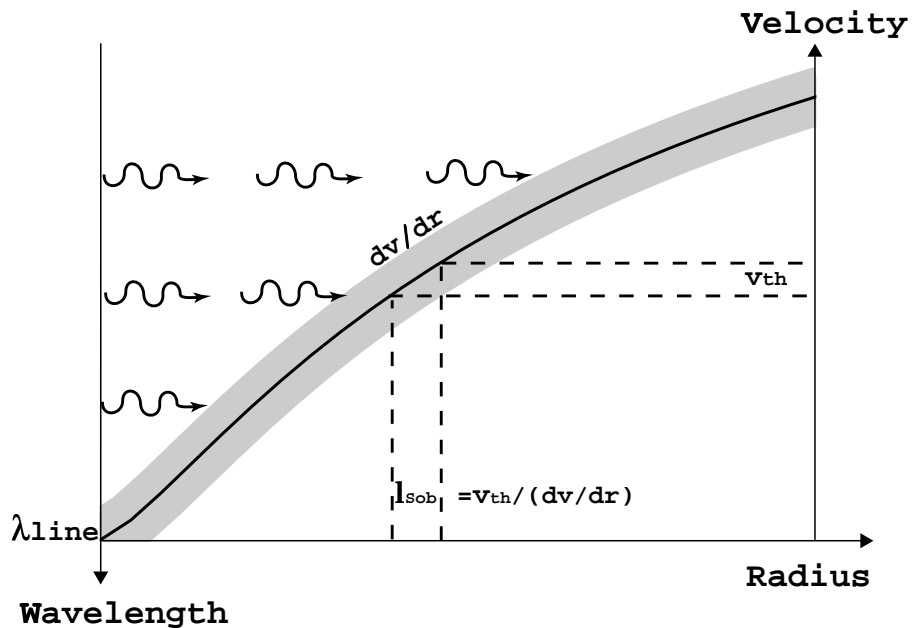
When an electron is bound into one of the discrete energy levels of an atom, its scattering of radiation is primarily with photons of just the right energy to induce shuffling with another discrete level. The process is called line-scattering, because it often results in the appearance of narrowly defined lines in a star's energy spectrum. At first glance, it may seem unlikely that such line-scattering could be effective in driving mass loss, simply because the opacity only interacts with a small fraction of the available stellar flux. But there are two key factors that work to make line-scattering in fact the key driving mechanism for hot-star winds.

The first is the *resonant* nature of line-scattering. The binding of an electron into discrete energy levels of an atom represents a kind of resonance cavity that can greatly amplify the interaction cross section with photons of just the right energy to induce transition among the levels. The effect is somewhat analogous to blowing into a whistle vs. just into open air. Like the sound of whistle, the response occurs at a well-tuned frequency, and has a greatly enhanced strength. Relative to free-electron scattering, the overall amplification factor for a broad-band, untuned radiation source is set by the *quality* of the resonance,  $Q \approx \nu_o/A$ , where  $\nu_o$  is the line frequency and  $A$  is decay rate of the excited state. For quantum mechanically allowed atomic transitions, this can be very large, of order  $10^7$ . Thus, even though only a very small fraction ( $\sim 10^{-4}$ ) of electrons in a hot-star atmosphere are bound into atoms, illumination of these atoms by an

*unattenuated* (i.e., optically thin), broad-band radiation source would yield a collective line-force that exceeds that from free electrons by about a factor  $\bar{Q} \approx 10^7 \times 10^{-4} = 1000$ . For stars within a factor two of the free-electron Eddington limit, this implies that line-scattering is capable, in principle, of driving material outward with an acceleration on order a *thousand* times the inward acceleration of gravity!

In practice, of course, this does not normally occur, since any sufficiently large collection of atoms scattering in this way would quickly block the limited flux available within just the narrow frequency bands tuned to the lines. Indeed, in the static portion of the atmosphere, the flux is greatly reduced at the line frequencies. Such line “saturation” keeps the overall line force quite small, in fact well below the gravitational force, which thus allows the inner parts of the atmosphere to remain gravitationally bound.

This, however, is where the second key factor, the *Doppler effect*, comes into play. In the outward-*moving* portions of the outer atmosphere, the Doppler effect red-shifts the local line resonance, effectively desaturating the lines by allowing the atoms to resonate with relatively unattenuated stellar flux that was initially at slightly higher frequencies. By effectively sweeping a broader range of the stellar flux spectrum, this makes it possible for the line force to overcome gravity and accelerate the very outflow it itself requires. As quantified within the CAK wind theory described later, the amount of mass accelerated adjusts such that the self-absorption of the radiation reduces the overall line-driving to being just somewhat (not a factor thousand) above what’s needed to overcome gravity.



Doppler-shifted line-resonance absorption in an accelerating flow. Photons with a wavelength just shortward of a line propagate freely from the stellar surface until redshifted, within a reference frame moving with the wind-outflow velocity, into a narrow line-resonance, whose width is set by the thermal broadening of the line, as represented here by the shading.

## THE SOBOLEV APPROXIMATION FOR LINE-SCATTERING IN AN ACCELERATING FLOW

The late Russian astrophysicist V. V. Sobolev developed an extremely useful approach for treating line-scattering in such a rapidly accelerating flow. As illustrated in figure 2, he noted how radiation emitted from the star at wavelength somewhat blueward of a line propagates freely until it is redshifted by the accelerating flow into a local line resonance. For the usual case that the broadening of the line is set by the ion thermal speed  $v_{th}$ , the geometric width of this resonance is about a *Sobolev length*,  $l_{Sob} \equiv v_{th}/(dv/dr)$ . In a supersonic flow, this Sobolev length is of order  $v_{th}/v \ll 1$  smaller than a typical flow variation scale, like the density/velocity scale length  $H \equiv |\rho/(d\rho/dr)| \approx v/(dv/dr)$ .

The nearly homogeneous conditions within such resonance layers imply that the key parameters of the line-scattering can be described in terms of strictly *local* conditions at any radius. In particular, the total optical depth of radiation passing through the line resonance – which normally requires evaluation of a nonlocal spatial integral – can in this case be well approximated simply in terms of the local density and velocity gradient,

$$\tau = \frac{\kappa \rho v_{th}}{dv/dr}, \quad (1)$$

where  $\kappa$  is the line opacity. This allows a localized solution for how much the flow absorption reduces the illumination of the line-resonance by the stellar flux, and leads to a simple, general expression for the line acceleration,

$$g_{line} \approx g_{thin} \frac{1 - e^{-\tau}}{\tau}. \quad (2)$$

In the optically thin limit  $\tau \ll 1$ , the line-acceleration reduces to a form similar to the electron scattering case,

$$g_{thin} \equiv \frac{\kappa v_{th} \nu_o L_\nu}{4\pi r^2 c^2} \approx \frac{\kappa}{\kappa_e} \frac{v_{th}}{c} g_e, \quad (3)$$

wherein the last equality applies the usual assumption that the line frequency  $\nu_o$  is near the peak of the stellar luminosity spectrum  $L_\nu$ , so that  $\nu_o L_\nu \approx L$ . In the opposite limit of an optically thick line with  $\tau \gg 1$ , there results a quite different form,

$$g_{thick} \approx \frac{g_{thin}}{\tau} = \frac{L}{4\pi r^2 \rho c^2} \frac{dv}{dr} = \frac{L}{\dot{M} c^2} v \frac{dv}{dr}, \quad (4)$$

where the last equality uses the definition of the wind mass loss rate,  $\dot{M} \equiv 4\pi \rho v r^2$ .

A key result here is that the optically thick line force is *independent* of the opacity, and instead varies in proportion to the velocity gradient  $dv/dr$ . The basis of this result is illustrated by figure 2, which shows that the local *rate* at which stellar radiation is red-shifted into a line-resonance depends on the *slope* of the velocity. By Newton's famous equation of motion, a force is normally understood to *cause* an acceleration. But here we see that an optically thick line-force also *depends* on the wind's advective rate of acceleration,  $v dv/dr$ .

## DERIVATION OF DIRECT-ABSORPTION FORCE FOR A SINGLE LINE

Let us derive this key property of line-driving in more quantitative terms. Under the simplifying approximation that the stellar radiation flux is purely radial (as from a central point source), the force-per-unit-mass associated with direct absorption by a single line at a radius  $r$  is given by

$$g_{line}(r) = g_{thin} \int_{-\infty}^{\infty} dx \phi(x - v(r)/v_{th}) e^{-t(x,r)}. \quad (5)$$

The integration is over a scaled frequency  $x \equiv (\nu/\nu_o - 1)(c/v_{th})$ , defined from line center in units of the frequency broadening associated with the ion thermal motion. The integrand is weighted by the line-profile function  $\phi(x)$ , which for thermal broadening typically has the Gaussian form  $\phi(x) \sim e^{-x^2}$ . The exponential reduction takes account of absorption, as set by the frequency-dependent optical depth to the stellar surface radius  $R_*$ ,

$$t(x, r) \equiv \int_{R_*}^r dr' \kappa \rho(r') \phi(x - v(r')/v_{th}).$$

A crucial point in evaluating this integral is that in a supersonic wind, the variation of the integrand is dominated by the velocity variation within the line-profile. As noted above, this variation has a scale given by the Sobolev length  $l_{Sob} \equiv v_{th}/(dv/dr)$ , which is smaller by a factor  $v_{th}/v$  than the competing density/velocity scale,  $H \equiv |\rho/(d\rho/dr)| \approx v/(dv/dr)$ . A key step in the Sobolev approximation is thus to recast this spatial integration as an integration over the comoving-frame frequency  $x' \equiv x - v(r')/v_{th}$ ,

$$t(x, r) = - \int_{x'(R_*)}^{x'(r)} \frac{dx' v_{th}}{dv/dr'} \kappa \rho(x') \phi(x') \approx \tau(r) \int_{x-v(r)/v_{th}}^{\infty} dx' \phi(x'), \quad (6)$$

where the latter approximation uses the assumption that  $v(r) \gg v_{th}$  to formally extend the surface frequency  $x'(R_*)$  to infinity relative to the local resonance  $x'(r)$ . The quantity  $\tau(r)$ , which is just the local Sobolev optical thickness defined in eqn. (1), arises as a collection of spatial variables that are assumed to be nearly constant over the Sobolev resonance zone, and thus can be extracted outside the integral.

Finally, a remarkable, extra bonus from this approximation is that the resulting optical depth (6) now has precisely the form needed to allow analytic evaluation of the line-force integral (5), yielding directly the general expression given in eqn. (2).

## THE MASS LOSS RATE FROM INERTIAL FORCE BALANCE

We can actually estimate the wind mass loss rate from a simple consideration of how this line-force overcomes the wind inertia (for now ignoring gravity). For a single optically thick line near the peak of the stellar flux spectrum, the resulting steady state flow acceleration is given by,

$$v \frac{dv}{dr} = g_{line} \approx \frac{L}{\dot{M} c^2} v \frac{dv}{dr}.$$

Since the acceleration cancels from both sides, this simply implies  $\dot{M} \approx L/c^2$ , or that the total mass loss of the wind is roughly equal to just the mass loss associated with the stellar luminosity! In practice, hot-star winds are driven by not just one, but many optically thick lines. As long as these are independent of each other, the total resulting mass loss simply accumulates in proportion to the total number of thick lines,

$$\dot{M} \approx N_{thick} \frac{L}{c^2}. \quad (7)$$

For a wind with terminal speed  $v_\infty$ , each line sweeps through about a fraction  $v_\infty/c$  of the spectrum. This means that there can at most be about  $N_{thick} \approx c/v_\infty$  non-overlapping thick lines spread throughout the spectrum. This implies a single-scattering limit for the mass loss,

$$\dot{M} v_\infty < \frac{L}{c}.$$

The winds from stars of spectral types O and B are generally within this limit, but winds from Wolf-Rayet (WR) stars can exceed it by a factor of ten or more. As further discussed below, the driving of such WR winds thus requires a more intricate, multiline scattering process.

## THE CAK THEORY FOR LINE-DRIVEN WINDS

In practice, the number of thick lines depends itself on the mass loss rate, and so is not known *a priori*. Self-consistent solution of the wind properties is possible, however, through the formalism developed by Castor, Abbott, and Klein (CAK). A key simplification is to approximate the flux-weighted number distribution of lines as a power law in the line opacity  $\kappa$ ,

$$\frac{dN}{d\kappa} = \frac{1}{\kappa_o} \left( \frac{\kappa}{\kappa_o} \right)^{\alpha-2}$$

where the CAK power-law index satisfies  $0 < \alpha < 1$ , and  $\kappa_o$  is a normalization constant defined such that  $\kappa_o dN/d\kappa \equiv 1$ . [The latter is related to the commonly defined CAK constant  $k$  by  $k = \Gamma(\alpha)(v_{th}/c)(\kappa_o/\kappa_e)^{1-\alpha}/(1-\alpha)$ , where  $\Gamma$  denotes the complete gamma function.] The cumulative force from this line-ensemble can be obtained by an integration of the single-line result (2) over this opacity distribution. This yields a kind of “geometric mean” between the optically thin and thick forms [eqns. (3) and (4)] for a single line,

$$g_{cak} = \frac{g_{thin,o}}{\tau_o^\alpha}. \quad (8)$$

Here  $\tau_o$  and  $g_{thin,o}$  are respectively the Sobolev optical thickness and the optically-thin acceleration for a single, spectral-peak line of opacity  $\kappa_o$ . This



normalization opacity is given roughly by the line-resonance amplification factor discussed previously,

$$\frac{\kappa_o v_{th}}{c} \approx \bar{Q}\kappa_e \approx 10^3 \kappa_e,$$

which thus implies that  $g_{thin,o} \approx 10^3 g_e$ , as noted earlier. In this context,  $\bar{Q}$  can be thus thought of an alternative, dimensionless way to specify the opacity normalization.

Ignoring the generally unimportant contribution of gas pressure, the steady-state equation of motion simply requires that the wind acceleration must equal the line-acceleration minus the inward acceleration of gravity,

$$v \frac{dv}{dr} = g_{cak} - \frac{GM(1 - \Gamma_e)}{r^2},$$

wherein we've also taken into account the effective reduction of gravity by the free-electron scattering factor  $\Gamma_e$ .

Defining the flow acceleration in units of the gravity as

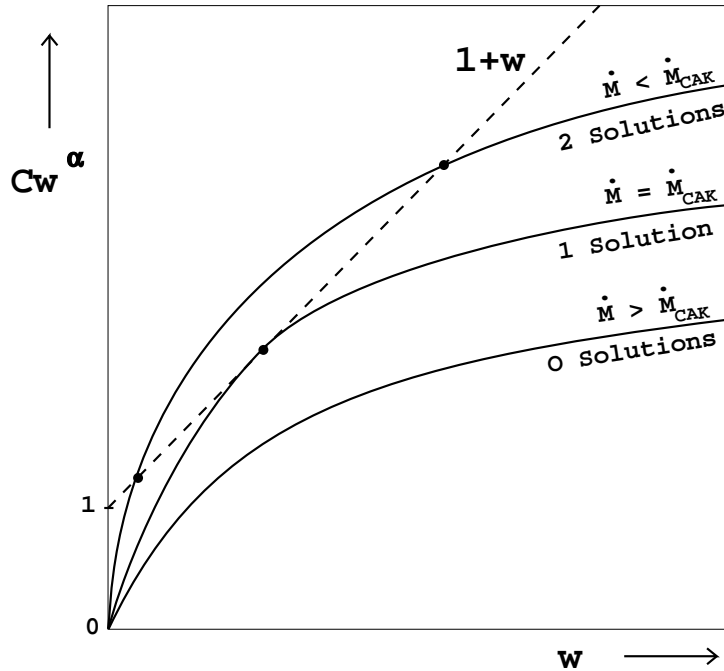
$$w \equiv r^2 v v' / GM(1 - \Gamma_e),$$

the equation of motion can be rewritten in the simple, dimensionless form,

$$w = C w^\alpha - 1, \quad (9)$$

where the constant is given by

$$C = \left( \frac{\bar{Q}\Gamma_e}{1 - \Gamma_e} \right)^{1-\alpha} \left( \frac{L}{Mc^2} \right)^\alpha.$$



**Figure 3.** Graphical solutions of the dimensionless equation of motion (9).

Figure 3 illustrates the graphical solution of this dimensionless equation of motion for various values of the constant  $C$ . For fixed stellar and opacity-distribution parameters, this corresponds to assuming various values of the mass loss rate  $\dot{M}$ . For high  $\dot{M}$ , there are no solutions, while for low  $\dot{M}$ , there are two solutions. The two limits are separated by a critical case with one solution – corresponding to the *maximal* mass loss rate – for which the function  $Cw^\alpha$  intersects the line  $1+w$  at a tangent. For this critical case, the tangency requirement implies  $\alpha C_c w_c^{\alpha-1} = 1$ , which together with the original equation (9) yields the critical conditions  $w_c = \alpha/1 - \alpha$  and  $C_c = 1/\alpha^\alpha (1 - \alpha)^{1-\alpha}$ .

Since eqn. (9) has no explicit spatial dependence, these conditions hold at all radii. By spatial integration of the critical acceleration  $w_c$  from the surface radius  $R_*$ , we thus obtain the CAK velocity law,

$$v(r) = v_\infty \left(1 - \frac{R_*}{r}\right)^{1/2} \quad (10)$$

where the terminal speed is given by  $v_\infty = v_{esc} \sqrt{\alpha/1 - \alpha}$ , where  $v_{esc}$  is the effective surface escape speed. Likewise, the associated critical value of  $C_c$  defines the maximal CAK mass loss rate

$$\dot{M}_{cak} = \alpha \left[ \frac{(1 - \alpha) \bar{Q} \Gamma_e}{1 - \Gamma_e} \right]^{\frac{1-\alpha}{\alpha}} \frac{L}{c^2} \quad (11)$$

Comparison with eqn. (7) shows that the first two factors in eqn. (11) now provide an explicit expression for the number of thick lines.

These results strictly apply only under the idealized assumption made above that the stellar radiation is radially streaming from a point-source. If one takes into account the finite angular extent of the stellar disk, then the mass loss is slightly reduced (typically by about a factor two), and the velocity law becomes slightly flatter (approximated by replacing the exponent 1/2 in eqn. (10) by a value  $\beta \approx 0.8$ ), with a somewhat higher terminal speed ( $v_\infty \approx 3v_{esc}$ ).

Finally, another simplifying assumption of this analysis is that the line-opacities are spatially constant, which implies a fixed wind ionization. The effect of a radial change in ionization can be approximately taken into account by correcting the CAK force (8) by a factor of the form  $(n_e/W)^\delta$ , where  $n_e$  is the electron density,  $W \equiv 0.5 \left(1 - \sqrt{1 - R_*/r}\right)$  is the radiation “dilution factor”, and the exponent has a typical value  $\delta \approx 0.1$ . This factor introduces an additional density dependence to that already implied by the optical depth factor  $1/\tau_o^\alpha$  given in eqn. (8). Its general effect on the mass loss can be roughly accounted with the simple substitution  $\alpha \rightarrow \alpha' \equiv \alpha - \delta$  in the power exponents of the CAK mass loss scaling law (11). The general tendency is to moderately increase  $\dot{M}$ , and accordingly to somewhat decrease the wind speed.

## THE WIND-MOMENTUM-LUMINOSITY RELATION

An important success of these CAK scaling laws is the theoretical explanation they provide for an empirically observed “Wind-Momentum-Luminosity” (WML) relation. Combining the CAK mass-loss law (11) together with the scaling of the terminal speed  $v_\infty$  with the effective escape speed  $v_{esc} = \sqrt{2GM(1-\Gamma)/R}$ , we obtain a WML relation of the form,

$$\dot{M}v_\infty\sqrt{R_*} \sim L^{1/\alpha'} Q^{1/\alpha'-1}$$

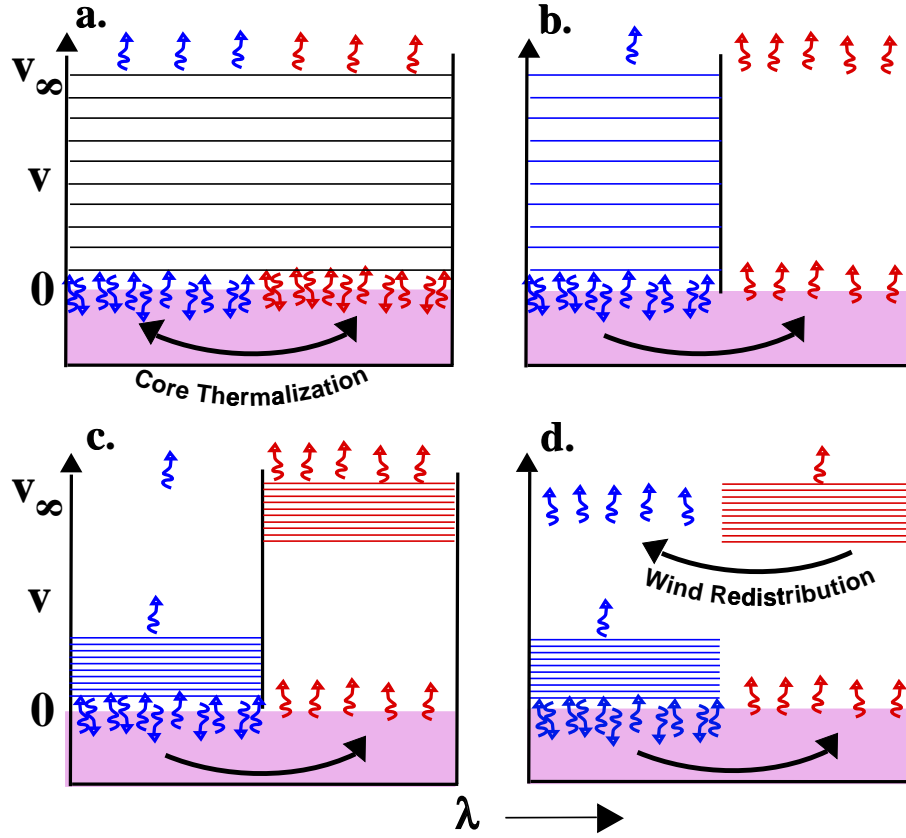
wherein we have neglected a residual dependence on  $M(1-\Gamma)$  that is generally very weak for the usual case that  $\alpha'$  is near  $2/3$ . Stellar mass loss rates can be routinely derived by fits to the observed Balmer ( $H\alpha$ ) line emission, and wind terminal speeds can be inferred from the blue edge of observed P-Cygni-profiles. Combined with spectroscopic estimates of the stellar luminosity and radius (the latter of which in any case enters only weakly as a square root), empirical fits for a large sample of galactic OB supergiants have demonstrated a remarkably tight agreement with a single WML relation. These fits give a luminosity slope that implies  $\alpha' \approx 0.57$ , and an overall normalization that is quite consistent with the expectation that  $\bar{Q} \approx 10^3$ .

Less extensive fits have also been derived for OB stars in the Magellanic clouds. These are each separately consistent with a single WML relation, but with a lower normalization, especially for the small cloud. This likely just reflects the weaker radiative driving from the lower metallicity  $Z$ , since the line opacity normalization varies in direct proportion to the metallicity,  $\bar{Q} \sim Z$ . There are currently efforts toward carefully calibrating this WML relation and its dependence on metallicity. The ultimate goal is to apply observationally inferred wind parameters of luminous hypergiant stars in external galaxies as an alternative “standard candle” for extragalactic distance measurements. In this way hot-star winds may play a role in calibrating the distance scale of the universe!

## THE WOLF-RAYET WIND “MOMENTUM PROBLEM”

Wolf-Rayet (WR) stars are evolved, massive, hot stars for which the cumulative mass loss has led to depletion of the original hydrogen envelope. They typically show broad wind-emission lines of elements like carbon, nitrogen, and/or oxygen that are the products of core nucleosynthesis. Overall, observations indicate that WR winds are especially strong, and even optically thick to continuum scattering by electrons. Notably, inferred WR wind momenta  $\dot{M}v_\infty$  are generally substantially higher than for OB stars of comparable luminosity, placing them well above the OB-star line in the WML relation. In fact, the inferred ratio of the wind momentum to that of stellar luminosity,  $\eta \equiv \dot{M}v_\infty/(L/c)$ , is typically much above unity, sometimes as high as  $\eta = 10 - 50$ !

This last property has generally been cast as the WR wind “momentum problem”, sometimes with the implication that it implies that WR winds cannot be radiatively driven. In fact, it merely means that, unlike for OB stars, WR winds cannot be treated in the standard, *single-scattering* formalism, which assumes optically thick lines do not overlap within the wind. But momentum ratios above unity can, in principle, be achieved by *multiple scattering* between overlapping thick lines with a velocity-unit frequency separation  $\Delta v < v_\infty$ . Indeed, if the spectral distribution of lines is “gray” – i.e., spread randomly through the spectrum without substantial gaps or bunches – then



Schematic diagram illustrating the role of line gaps, line bunches, and photon thermalization for WR wind momentum deposition. The horizontal lines represent the velocity/frequency spacing of optically thick lines in the wind. The four parts represent: (a.) an effectively gray model (b.) a wind with fixed ionization and extensive gaps (c.) a wind with ionization stratification that fills gaps (d.) the importance of limiting energy redistribution in the wind.

the momentum attained scales simply with the mean thick-line velocity separation as  $\eta \approx v_\infty / \Delta v$  (figure 4a).

However, realistic line lists are not gray, and leakage through gaps in the line spectral distribution tends to limit the effective scattering to  $\eta \lesssim 1$  (figure 4b). But the optically thick nature of the WR winds gives them a substantial ionization stratification, and this helps spread line-bunches and so fill in gaps, allowing for more effective global trapping of radiation that makes  $\eta > 1$  (figure 4c). Redistribution of photon energy can reduce the local effectiveness of line-driving (figure 4d). In particular, redistribution by photon thermalization near the stellar core makes it especially difficult for radiation alone to initiate the wind. The relative complexity of WR wind initiation may be associated with the extensive turbulent structure inferred from observed variability in WR wind emission lines. Overall, the understanding of WR winds is perhaps best viewed as an “opacity problem”, i.e. identifying the enhanced opacity that can adequately block the radiation flux throughout the wind, and thus drive a WR mass loss that is much greater than from OB stars of comparable luminosity.

## WIND INSTABILITY AND VARIABILITY

There is extensive evidence that hot-star winds are not the smooth, steady outflows idealized in the simple CAK theory, but rather have extensive structure and variability on a variety of scales. The most direct example is the explicit variability often detected in unsaturated absorption troughs of P-Cygni line-profiles. To produce such explicit line-profile variability, the associated wind structure must be relatively large-scale, covering a substantial fraction of the stellar disk. (By comparison, the emission component of P-Cygni profiles is generally much less variable, since it forms from a more global average of the wind.)

Much of the variability can be characterized as “Discrete Absorption Components”. These begin as broad absorption enhancement in the inner part of the blue absorption trough of an unsaturated P-Cygni line, which then gradually narrow as they drift, over a period of days, toward the blue-edge of the profile. Their occurrence is generally irregular, and their apparent acceleration is quite slow, much slower than expected for an element of the steady wind. They may reflect the slow acceleration of mass ejected by some kind of semi-random disturbance from the underlying stellar surface. The exact nature of this is unknown, but might possibly be linked to either magnetic activity or stellar pulsation.

In a second class of explicit variation are the “Periodic Absorption Modulations”. These occur at regular, sometimes multiple periods, and include both a modulated increase and decrease in the absorption strength. The periods are generally of order a few days, which is consistent with a low-order multiple of the stellar rotation period. They likely represent rotational modulation of wind structure by an unknown kind of surface irregularity (again perhaps magnetic).

The optical emission lines formed in Wolf-Rayet winds also commonly show a low-amplitude variability. Such optical lines have been monitored using ground-based telescopes at much higher signal-to-noise than has been typical for the ultraviolet wind lines monitored from OB stars by orbiting satellites. Analysis suggests a turbulent hierarchy of structure extending down to quite small spatial scales.

There is also indirect evidence that OB winds have an extensive, small-scale, turbulent structure. Saturated P-Cygni lines have extended black troughs thought to be a signature that the wind velocity is highly nonmonotonic. Such stars commonly show soft X-ray emission and sometimes also non-thermal radio emission, both of which are thought to originate from embedded wind shocks.

A promising explanation for this small-scale, turbulent structure is the intrinsically strong instability of line-driving to small-scale velocity perturbations. As noted above, there is a strong hidden potential in line-scattering to drive wind material with accelerations that greatly exceed the mean outward acceleration. Numerical simulations suggest this can indeed lead naturally to a highly structured flow dominated by multiple shock compressions. But it is not yet clear whether the simulated flow properties can robustly explain key observational features like the soft X-ray emission.

The theory for line-driven winds is mature enough that it is now finding application in many more complex circumstances. Let us thus briefly summarize some of these frontiers.

One example is colliding winds of close, massive star binaries. In several cases these involve WR+O stars with separations of only a few O-star radii. Generally the WR wind is so much stronger that a simple hydrodynamic ram balance between the two stars is not possible. This suggests the WR wind should simply overwhelm its companion's outflow, and penetrate down to the O-star surface. Orbital phase monitoring of such systems suggest, however, that this does not occur, but that a wind-wind interface is generally kept away from the O-star surface. An attractive explanation for this comes from considering the additional pressure provided by the O-star light in a *radiative braking* of the impacting WR wind. Such a radiative deceleration has many of the same properties of the usual wind acceleration, including a crucial role for the Doppler shift of line-resonance in allowing a strong line-force. An interesting result is that radiative braking is generally only effective if the interaction of O-star light WR material is characterized by a stronger line-opacity than needed for driving the O-star wind. This represents a new twist to the usual "opacity problem" of how WR stars drive such strong winds, and even suggests that observations of WR+O binaries could help constrain answers to this fundamental question.

Hot-stars generally have a rapid rotation, but among the fastest rotators are the Be stars, which are characterized by strong Balmer line emission thought to originate in a circumstellar disk. A quite appealing idea is that these form naturally as "Wind Compressed Disks" by rotational focussing of the star's radiatively driven stellar wind toward the equatorial plane. Initial dynamical simulations that assumed a simple, radial form for the line-force generally supported this idea. Surprisingly, however, subsequent, more realistic models show that *nonradial* components of this line-force can actually completely *inhibit* the formation of such disk by effectively driving material away from the equatorial plane. Further analysis has shown how this, and other peculiar properties of line-driving, follows naturally from the characteristic scaling of the line-force with the velocity gradient.

Finally, line-driven winds can also originate from luminous accretion disks, such as those found in cataclysmic variables. Recent work has thus focussed on extending the basic CAK theory to account for key differences from the stellar case, for example the 2D nature of the flow, and the peculiar nonmonotonic variation of the effective gravity, which initially increases from the disk plane before eventually falling off with distance. There is even renewed interest in developing such disk wind models to explain the broad-line flows observed from active galactic nuclei and quasars!

## BIBLIOGRAPHY

The theory of hot-star winds is nearly three decades old, and in this time there has amassed an appropriately large literature. The following provides an incomplete list of some key papers, and points to some reviews.

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