15 SURVEYING THE STARS

LEARNING GOALS

15.1 PROPERTIES OF STARS

- How do we measure stellar luminosities?
- How do we measure stellar temperatures?
- How do we measure stellar masses?

15.2 PATTERNS AMONG STARS

- What is a Hertzsprung-Russell diagram?
- What is the significance of the main sequence?
- What are giants, supergiants, and white dwarfs?
- Why do the properties of some stars vary?

15.3 STAR CLUSTERS

- What are the two types of star clusters?
- How do we measure the age of a star cluster?

"All men have the stars," he answered, "but they are not the same things for different people. For some, who are travelers, the stars are guides. For others they are no more than little lights in the sky. For others, who are scholars, they are problems. For my businessman they were wealth. But all these stars are silent. You—you alone—will have the stars as no one else has them."

—Antoine de Saint-Exupéry, from *The Little Prince*

On a clear, dark night, a few thousand stars are visible to the naked eye. Many more become visible through binoculars, and a powerful telescope reveals so many stars that we could never hope to count them. Like each individual person, each individual star is unique. Like all humans, all stars have much in common.

Today, we know that stars are born from clouds of interstellar gas, shine brilliantly by nuclear fusion for millions to billions of years, and then die, sometimes in dramatic ways. In this chapter, we'll discuss how we study and categorize stars and how we have come to realize that stars, like people, change over their lifetimes.

15.1 PROPERTIES OF STARS

Imagine that an alien spaceship flies by Earth on a simple but short mission: The visitors have just 1 minute to learn everything they can about the human race. In 60 seconds, they will see next to nothing of any individual person's life. Instead, they will obtain a collective "snapshot" of humanity that shows people from all stages of life engaged in their daily activities. From this snapshot alone, they must piece together their entire understanding of human beings and their lives, from birth to death.

We face a similar problem when we look at the stars. Compared with stellar lifetimes of millions or billions of years, the few hundred years humans have spent studying stars with telescopes is rather like the aliens' 1-minute glimpse of humanity. We see only a brief moment in any star's life, and our collective snapshot of the heavens consists of such frozen moments for billions of stars. From this snapshot, we try to reconstruct the life cycles of stars.

Thanks to the efforts of hundreds of astronomers studying this snapshot of the heavens, stars are no longer mysterious points of light in the sky. We now know that all stars have much in common with the Sun. They all form in great clouds of gas and dust, and each one begins its life with roughly the same chemical composition as the Sun: About three-quarters of a star's mass at birth is hydrogen, and about one-quarter is helium, with no more than about 2% consisting of elements heavier than helium. Nevertheless, stars are not all the same; they differ in such properties as size, age, brightness, and temperature. We'll devote most of this and the next two chapters to understanding how and why stars differ. First, however, let's explore how we measure three of the most fundamental properties of stars: luminosity, surface temperature, and mass.

How do we measure stellar luminosities?

If you go outside on any clear night, you'll immediately see that stars differ in brightness. Some stars are so bright that we can use them to identify constellations [Section 2.1]. Others are so dim that our naked eyes cannot see them at all. However, these differences in brightness do not by themselves tell us anything about how much light these stars are generating, because the brightness of a star depends on its distance as well as on how much light it actually emits. For example, the stars Procyon and Betelgeuse, which make up two of the three corners of the Winter Triangle (see Figure 2.2), appear about equally bright in our sky. However, Betelgeuse actually emits about 5000 times as much light as Procyon. It has about the same brightness in our sky because it is much farther away.

SEE IT FOR YOURSELF

Until the 20th century, people classified stars primarily by their brightness and location in our sky. On the next clear night, find a favorite constellation and visually rank the stars by brightness. Then look to see how that constellation is represented on the star charts in Appendix I. Why do the star charts use different size dots for different stars? Do the brightness rankings on the star chart agree with what you see?

Because two similar-looking stars can be generating very different amounts of light, we need to distinguish clearly between a star's brightness in our sky and the actual amount of light that it emits into space (Figure 15.1):

- When we talk about how bright stars look in our sky, we are talking about **apparent brightness**—the brightness of a star as it appears to our eyes. We define the apparent brightness of any star in our sky as the amount of power (energy per second) reaching us *per unit area.* (A more technical term for apparent brightness is *flux.*)

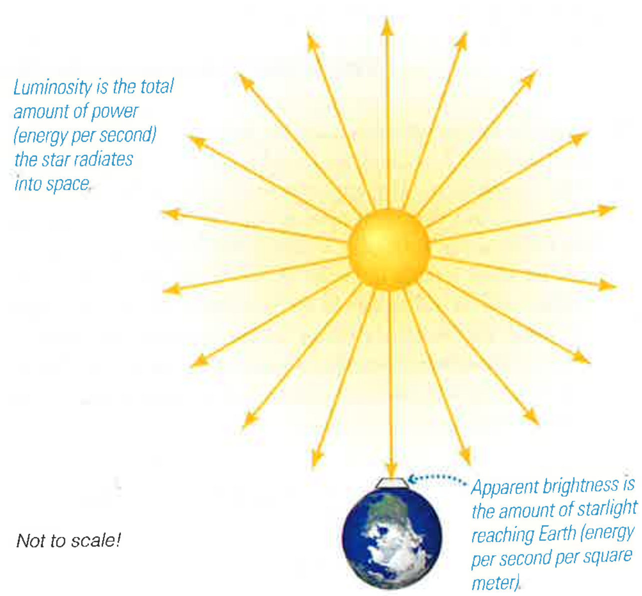


FIGURE 15.1 Luminosity is a measure of power, and apparent brightness is a measure of power per unit area.

When we talk about how bright stars are in an absolute sense, regardless of their distance, we are talking about **luminosity**—the total amount of power that a star emits into space.

We can understand the difference between apparent brightness and luminosity by thinking about a 100-watt light bulb. The bulb always puts out the same amount of light, so its luminosity doesn't vary. However, its apparent brightness depends on your distance from the bulb: It will look quite bright if you stand very close to it, but quite dim if you are far away.

The Inverse Square Law for Light The apparent brightness of a star or any other light source obeys an *inverse square law* with distance, much like the inverse square law that describes the force of gravity [Section 4.4]. For example, if we viewed the Sun from twice Earth's distance, it would appear dimmer by a factor of $2^2 = 4$. If we viewed it from 10 times Earth's distance, it would appear dimmer by a factor of $10^2 = 100$.

Figure 15.2 shows why apparent brightness follows an inverse square law. The same total amount of light must pass through each imaginary sphere surrounding the star. If we focus on the light passing through the small square on the sphere located at 1 AU, we see that the same amount of light must pass through *four* squares of the same size on the sphere located at 2 AU. Each square on the sphere at 2 AU therefore receives only $\frac{1}{2^2} = \frac{1}{4}$ as much light as the square on the sphere at 1 AU. Similarly, the same amount of light passes through *nine* squares of the same size on the sphere located at 3 AU, so each of these squares receives only $\frac{1}{3^2} = \frac{1}{9}$ as much light as the square on the sphere at 1 AU. Generalizing, the amount of light received per unit

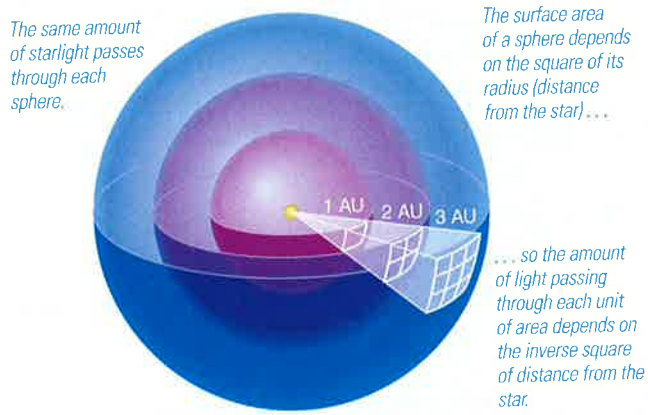


FIGURE 15.2 The inverse square law for light: The apparent brightness of a star declines with the square of its distance.

area decreases with increasing distance by the square of the distance—an inverse square law.

This inverse square law leads to a very simple and important formula relating the apparent brightness, luminosity, and distance of any light source. We will call it the **inverse square law for light**:

$$\text{apparent brightness} = \frac{\text{luminosity}}{4\pi \times \text{distance}^2}$$

Because the standard units of luminosity are watts [Section 14.1], the units of apparent brightness are *watts per square meter*. (The 4π in the formula above comes from the fact that the surface area of a sphere is given by $4\pi \times \text{radius}^2$.)

In principle, we can always determine a star's apparent brightness by carefully measuring the amount of light we receive from the star per square meter. We can then use the inverse square law to calculate a star's luminosity if we can first measure its distance, or to calculate a star's distance if we somehow know its luminosity.

MATHEMATICAL INSIGHT 15.1

The Inverse Square Law for Light

We can derive the inverse square law for light by extending the idea illustrated in Figure 15.2. Suppose we are located a distance d from a star with luminosity L. The apparent brightness of the star is the power per unit area that we receive at our distance, d. We find this apparent brightness by imagining a giant sphere with radius d (similar to any of the three spheres in Figure 15.2) and surface area $4\pi \times d^2$. (The surface area of any sphere is $4\pi \times \text{radius}^2$.) All the star's light passes through the imaginary sphere, so the apparent brightness at any point on this sphere is simply the star's luminosity, L, divided by the sphere's surface area; carrying out the division gives us the inverse square law for light:

$$\text{apparent brightness} = \frac{\text{star's luminosity}}{\text{surface area of imaginary sphere}} = \frac{L}{4\pi \times d^2}$$

EXAMPLE: What is the Sun's apparent brightness in our sky?

SOLUTION:

Step 1 Understand: The Sun's apparent brightness is the power per unit area that we receive in the form of sunlight. We find this power with the inverse square law for light, using the Sun's luminosity and Earth's distance from the Sun; for unit consistency, we put the Earth-Sun distance in meters.

Step 2 Solve: The Sun's luminosity is $L_{Sun} = 3.8 \times 10^{26}$ watts, and Earth's distance from the Sun is $d = 1.5 \times 10^{11}$ meters. The Sun's apparent brightness is therefore

$$\frac{L}{4\pi \times d^2} = \frac{3.8 \times 10^{26}\ \text{watts}}{4\pi \times (1.5 \times 10^{11}\ \text{m})^2} = 1.3 \times 10^3\ \text{watts/m}^2$$

Step 3 Explain: The Sun's apparent brightness is about 1300 watts per square meter at Earth's distance. This is the maximum power per unit area that could be collected by a detector on Earth that directly faces the Sun, such as a solar power (or *photovoltaic*) cell.

THINK ABOUT IT

Suppose Star A is four times as luminous as Star B. How will their apparent brightnesses compare if they are both the same distance from Earth? How will their apparent brightnesses compare if Star A is twice as far from Earth as Star B? Explain.

Measuring Apparent Brightness We can measure a star's apparent brightness by using a detector, such as a CCD, that records how much energy strikes its light-sensitive surface each second. For example, such a detector would record an apparent brightness of 2.7×10^{-8} watt per square meter from Alpha Centauri A (the brightest of the three stars in the Alpha Centauri system). The only difficulties we face in measuring apparent brightness are making sure the detector is properly calibrated and, for ground-based telescopes, taking into account the absorption of light by Earth's atmosphere.

No detector can record light of all wavelengths, so we necessarily measure apparent brightness in only some small range of the complete spectrum. For example, the human eye is sensitive to visible light but does not respond to ultraviolet or infrared photons. When we perceive a star's brightness, our eyes are measuring the apparent brightness only in the visible region of the spectrum.

When we measure the apparent brightness in visible light, we can calculate only the star's *visible-light luminosity*. Similarly, when we observe a star with a spaceborne X-ray telescope, we measure only the apparent brightness in X rays and can calculate only the star's *X-ray luminosity*. We will use the terms **total luminosity*** and **total apparent brightness** to describe the luminosity and apparent brightness we would measure *if* we could detect photons across the entire electromagnetic spectrum.

One important complication often arises when we try to calculate a star's luminosity from its apparent brightness: The inverse square law for light works perfectly only if the starlight follows an uninterrupted path to Earth. In reality, the light of most stars passes through at least some clouds containing interstellar dust on its way to Earth, and this dust can absorb or scatter some of the star's light [Section 16.1]. Today, thanks largely to our modern scheme of stellar classification, we can usually measure the effect of interstellar dust and account for it when we apply the inverse square law for light. A century ago, before astronomers knew of the existence of interstellar dust, astronomers often underestimated stellar distances because they did not realize that the dust was making stars appear less bright than they really are.

MA Measuring Cosmic Distances Tutorial, Lesson 2

Measuring Distance Through Stellar Parallax The most direct way to measure a star's distance is with *stellar parallax*, the small annual shifts in a star's apparent position caused by Earth's motion around the Sun [Section 2.4]. Recall that you can observe parallax of your finger by holding it at arm's length and looking at it with first one eye closed and then the other. Astronomers measure stellar parallax

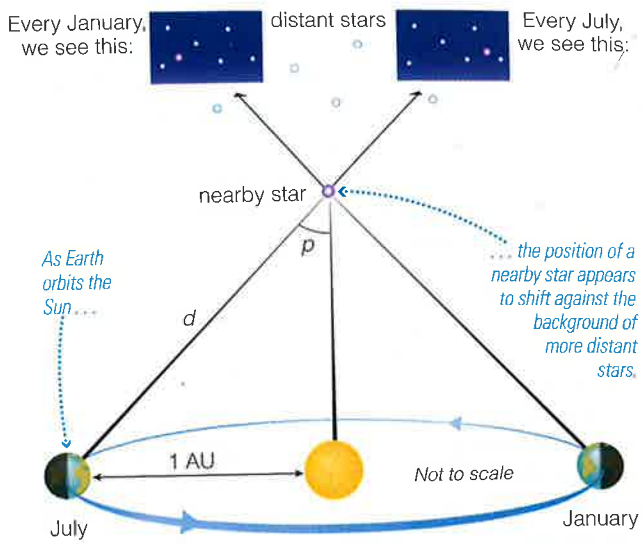


FIGURE 15.3 Interactive Figure Parallax makes the apparent position of a nearby star shift back and forth with respect to distant stars over the course of each year. The angle *p*, called the *parallax angle*, represents half the total parallax shift each year. If we measure *p* in arcseconds, the distance *d* to the star in parsecs is $1/p$. The angle in this figure is greatly exaggerated: All stars have parallax angles of less than 1 arcsecond.

by comparing observations of a nearby star made 6 months apart (Figure 15.3). The nearby star appears to shift against the background of more distant stars because we are observing it from two opposite points of Earth's orbit.

We can calculate a star's distance if we know the precise amount of the star's annual shift due to parallax. This means measuring the angle *p* in Figure 15.3, which we call the star's *parallax angle* and is equal to *half* the star's annual back-and-forth shift. Notice that this angle would be smaller if the star were farther away, so we conclude that more distant stars have smaller parallax angles.

All stars are so far away that they have very small parallax angles, which explains why the ancient Greeks were never able to measure parallax with their naked eyes. Even the nearest stars have parallax angles smaller than 1 arcsecond—well below the approximately 1 arcminute angular resolution of the naked eye [Section 6.2]. For increasingly distant stars, the parallax angles quickly become too small to measure even with our highest-resolution telescopes. Current technology allows us to measure parallax accurately only for stars within a few hundred light-years—not much farther than what we call our *local solar neighborhood* in the vast, 100,000-light-year-diameter Milky Way Galaxy.

By definition, the distance to an object with a parallax angle of 1 arcsecond is 1 **parsec (pc)**. (The word *parsec* comes from combining the words *parallax* and *arcsecond*.) Because all stars have parallax angles smaller than one arcsecond, they are all farther than 1 parsec away. If we use units of arcseconds for the parallax angle, *p*, a simple formula allows us to calculate distances in parsecs:

$$d \text{ (in parsecs)} = \frac{1}{p \text{ (in arcseconds)}}$$

For example, the distance to a star with a parallax angle of $\frac{1}{2}$ arcsecond is 2 parsecs, the distance to a star with a parallax

*Astronomers sometimes refer to the total luminosity as the *bolometric* luminosity.

angle of $\frac{1}{10}$ arcsecond is 10 parsecs, and the distance to a star with a parallax angle of $\frac{1}{100}$ arcsecond is 100 parsecs.

Astronomers often state distances in parsecs, kiloparsecs (1000 parsecs), or megaparsecs (1 million parsecs). However, with a bit of geometry, it's possible to show that 1 parsec is equivalent to 3.26 light-years (see Mathematical Insight 15.2). We can therefore modify the above formula slightly to give distances in light-years:

$$d \text{ (in light-years)} = 3.26 \times \frac{1}{p \text{ (in arcseconds)}}$$

In this book, we'll generally state distances in light-years rather than parsecs.

Parallax was the first reliable technique astronomers developed for measuring distances to stars, and it remains the only technique that tells us stellar distances without any assumptions about the nature of stars. If we know a star's distance from parallax, we can calculate its luminosity with the inverse square law for light. We now have parallax measurements for thousands of stars, which is a large enough number that astronomers have been able to draw some general conclusions about them. As we'll see later, these lessons have taught astronomers how to estimate luminosities for many more stars, even without knowing their distances. Astronomers today often use the inverse square law for light to calculate distances to objects for which we can reliably estimate luminosities, as well as to calculate luminosities of objects for which we have measured distances.

Parallax measurements have given us detailed knowledge of what our local solar neighborhood is like. For example, we know of more than 300 stars within about 33 light-years (10 parsecs) of the Sun. About half are binary star systems consisting of two orbiting stars, or multiple star systems containing three or more stars. Most are tiny, dim red stars—so dim that we cannot see them with the naked eye, despite the fact that they are relatively close. A few nearby stars, such as Sirius (8.6 light-years), Altair (17 light-years), Vega (25 light-years), and Fomalhaut (25 light-years), are white in color and bright in our sky, but most of the brightest stars in the sky lie farther away. Because so many nearby stars appear dim while many more distant stars appear bright, stellar luminosities must span a wide range.

The Luminosity Range of Stars Now that we have discussed how we determine stellar luminosities, it's time to take a quick look at the results; these results have been drawn both from stars for which we have parallax measurements and from those for which we determine distance in other ways. We usually state stellar luminosities in comparison to the Sun's luminosity, which we write as L_{Sun} for short. For example, Proxima Centauri, the nearest of the three stars in the Alpha Centauri system and hence the nearest star besides our Sun, is only about 0.0006 times as luminous as the Sun, or $0.0006L_{Sun}$. Betelgeuse, the bright left-shoulder star of Orion, has a luminosity of $38{,}000L_{Sun}$, meaning that it is

MATHEMATICAL INSIGHT 15.2

The Parallax Formula

We can derive the formula relating a star's distance and parallax angle by studying Figure 15.3. The parallax angle p is part of a right triangle, and from trigonometry you may recall that the *sine* of angle p is the length of the side opposite this angle divided by the length of the hypotenuse. Because the side opposite p is the Earth-Sun distance of 1 AU and the hypotenuse is the distance d to the object, we find

$$\sin p = \frac{\text{length of opposite side}}{\text{length of hypotenuse}} = \frac{1 \text{ AU}}{d}$$

Solving for d, the formula becomes

$$d = \frac{1 \text{ AU}}{\sin p}$$

By definition, 1 parsec is the distance to an object with a parallax angle of 1 arcsecond (1″), or $\frac{1}{3600}$ degree (because $1° = 60'$ and $1' = 60''$). Substituting these numbers into the parallax formula and using a calculator to find that $\sin 1'' = 4.84814 \times 10^{-6}$, we get

$$1 \text{ parsec} = \frac{1 \text{ AU}}{\sin 1''} = \frac{1 \text{ AU}}{4.84814 \times 10^{-6}} = 206{,}265 \text{ AU}$$

That is, 1 parsec = 206,265 AU. Converting units, we also find that 1 parsec $= 3.09 \times 10^{13}$ km = 3.26 light-years (because 1 AU = 149.6 million km and 1 light-year $= 9.46 \times 10^{12}$ km).

We need one more fact from geometry to derive the parallax formula given in the text. As long as the parallax angle, p, is small, $\sin p$ is proportional to p. For example, $\sin 2''$ is twice as large as $\sin 1''$, and $\sin \frac{1}{2}''$ is half as large as $\sin 1''$. (You can verify these examples with your calculator.) If we use $\frac{1}{2}''$ instead of 1″ for the parallax angle in the formula above, we get a distance of 2 parsecs instead of 1 parsec. Similarly, if we use a parallax angle of $\frac{1}{10}''$, we get a distance of 10 parsecs. Generalizing, we get the simple parallax formula given in the text:

$$d \text{ (in parsecs)} = \frac{1}{p \text{ (in arcseconds)}}$$

EXAMPLE 1: Sirius, the brightest star in our night sky, has a measured parallax angle of 0.379″. How far away is Sirius in parsecs? In light-years?

SOLUTION:

Step 1 Understand: We are given the parallax angle for Sirius in arcseconds, so we use the parallax formula to find its distance. Because the parallax angle is between 0.1″ and 1″, we expect the answer to be a distance between 1 and 10 parsecs.

Step 2 Solve: Substituting the parallax angle of 0.379″ into the formula, we find that the distance to Sirius in parsecs is

$$d \text{ (in parsecs)} = \frac{1}{0.379} = 2.64 \text{ pc}$$

Because 1 parsec = 3.26 light-years, this distance is equivalent to

$$2.64 \text{ parsecs} \times 3.26 \frac{\text{light-years}}{\text{parsec}} = 8.60 \text{ light-years}$$

Step 3 Explain: From its measured parallax angle, we have found that the distance to Sirius is 2.64 parsecs, or 8.60 light-years.

38,000 times as luminous as the Sun. Overall, studies of the luminosities of many stars have taught us two particularly important lessons:

- Stars have a wide range of luminosities, with our Sun somewhere in the middle. The dimmest stars have luminosities $\frac{1}{10,000}$ times that of the Sun ($10^{-4}L_{Sun}$), while the brightest stars are about 1 million times as luminous as the Sun ($10^6 L_{Sun}$).
- Dim stars are far more common than bright stars. For example, even though our Sun is roughly in the middle of the overall range of stellar luminosities, it is brighter than the vast majority of stars in our galaxy.

The Magnitude System The methods we've discussed for describing apparent brightness and luminosity work perfectly well, but many amateur and professional astronomers still describe these quantities in another way: They use the ancient **magnitude system** devised by the Greek astronomer Hipparchus (c. 190–120 B.C.).

The magnitude system originally classified stars according to how bright they look to human eyes, which were the only instruments available to measure brightness in ancient times. The brightest stars received the designation "first magnitude," the next brightest "second magnitude," and so on. The faintest visible stars were magnitude 6. We call these descriptions **apparent magnitudes** because they compare how bright different stars *appear* in the sky. Notice that apparent magnitudes are directly related to apparent brightness, except the scale runs backward: A larger apparent magnitude means a dimmer apparent brightness. For example, a star of magnitude 4 is dimmer in the sky than a star of magnitude 1. Star charts (such as those in Appendix I) often use dots of different sizes to represent the apparent magnitudes of stars. Larger dots represent brighter stars, which means those with smaller magnitude numbers.

In modern times, the magnitude system has been extended and more precisely defined. Each difference of five magnitudes is defined to represent a factor of exactly 100 in brightness. For example, a magnitude 1 star is 100 times as bright as a magnitude 6 star, and a magnitude 3 star is 100 times as bright as a magnitude 8 star. As a result of this precise definition, stars can have fractional apparent magnitudes and a few bright stars have apparent magnitudes *less than* 1—which means *brighter* than magnitude 1. For example, the brightest star in the night sky, Sirius, has an apparent magnitude of −1.46. Appendix F gives apparent magnitudes and actual luminosities for both the nearest stars and the brightest stars visible in the sky.

MATHEMATICAL INSIGHT 15.3

The Modern Magnitude Scale

The modern magnitude system is defined so that each difference of five magnitudes corresponds to a factor of exactly 100 in brightness. A single magnitude therefore corresponds to a factor of $(100)^{1/5} \approx 2.512$ in brightness. Given this fact, the following formula allows us to calculate the ratio of the apparent brightnesses of two stars from their apparent magnitudes:

$$\frac{\text{apparent brightness of Star 1}}{\text{apparent brightness of Star 2}} = (100^{1/5})^{m_2 - m_1}$$

where m_1 and m_2 are the apparent magnitudes of Stars 1 and 2, respectively.

If we replace the apparent magnitudes with absolute magnitudes (designated M instead of m), the same formula allows us to calculate the ratio of stellar luminosities:

$$\frac{\text{luminosity of Star 1}}{\text{luminosity of Star 2}} = (100^{1/5})^{M_2 - M_1}$$

EXAMPLE 1: On a clear night, stars dimmer than magnitude 5 are quite difficult to see. However, sensitive instruments on large telescopes can detect objects as faint as magnitude 30. How much more sensitive are such telescopes than the human eye?

SOLUTION:

Understand: We imagine that our eye sees "Star 1" with magnitude 5 and the telescope detects "Star 2" with magnitude 30. Because every five steps up in magnitude corresponds to a drop of a factor of 100 in apparent brightness, the apparent brightness of Star 2 must be far smaller than that of Star 1. In order to determine that difference in apparent brightness, we can use the first formula above.

Solve: Substituting $m_1 = 5$ and $m_2 = 30$ into the formula, we find

$$\frac{\text{apparent brightness of Star 1}}{\text{apparent brightness of Star 2}} = (100^{1/5})^{m_2 - m_1}$$
$$= (100^{1/5})^{30-5}$$
$$= (100^{1/5})^{25} = 100^5 = 10^{10}$$

Explain: The magnitude 5 star is 10^{10}, or 10 billion, times brighter than the magnitude 30 star, so the telescope is 10 billion times more sensitive than the human eye.

EXAMPLE 2: The Sun has an absolute magnitude of about 4.8. Polaris, the North Star, has an absolute magnitude of −3.6. How much more luminous is Polaris than the Sun?

SOLUTION:

Understand: Luminosity and absolute magnitude are two different ways of describing a star's total power output, and the second formula above gives the relationship between them. We can therefore use that formula with Polaris as Star 1 and the Sun as Star 2 to compare the luminosities of the two stars.

Solve: Substituting $M_1 = -3.6$ for Polaris and $M_2 = 4.8$ for the Sun into the formula, we find

$$\frac{\text{luminosity of Polaris}}{\text{luminosity of Sun}} = (100^{1/5})^{M_2 - M_1} = (100^{1/5})^{4.8-(-3.6)}$$
$$= (100^{1/5})^{8.4} = 100^{1.7} \approx 2500$$

Explain: From their absolute magnitudes, we have found that Polaris is about 2500 times as luminous as the Sun.

The modern magnitude system also defines **absolute magnitudes** as a way of describing stellar luminosities. A star's absolute magnitude is the apparent magnitude it would have *if* it were at a distance of 10 parsecs (32.6 light-years) from Earth. For example, the Sun's absolute magnitude is about 4.8, meaning that the Sun would have an apparent magnitude of 4.8 *if* it were 10 parsecs away from us—bright enough to be visible but not conspicuous on a dark night. Although many articles and books still quote apparent and absolute magnitudes, comparisons between stars are much easier when we think about how apparent brightness depends on luminosity according to the inverse square law. We'll therefore stick to the inverse square law in this book.

How do we measure stellar temperatures?

A second fundamental property of a star is its surface temperature. You might wonder why we emphasize *surface* temperature rather than interior temperature. The answer is that only surface temperature is directly measurable; interior temperatures are inferred from mathematical models of stellar interiors [Section 14.2]. Whenever you hear astronomers speak of the "temperature" of a star, you can be pretty sure they mean surface temperature unless they state otherwise.

Measuring a star's surface temperature is somewhat easier than measuring its luminosity, because the star's distance doesn't affect the measurement. Instead, we determine surface temperature from either the star's color or its spectrum. Let's briefly investigate how each technique works.

Color and Temperature Take a careful look at Figure 15.4. Notice that stars come in almost every color of the rainbow. Simply looking at the colors tells us something about the surface temperatures of the stars. For example, a red star is cooler than a yellow star, which in turn is cooler than a blue star.

Stars come in different colors because they emit thermal radiation [Section 5.4]. Recall that a thermal radiation spectrum depends only on the (surface) temperature of the object that emits it (see Figure 5.19). For example, the Sun's 5800 K surface temperature causes it to emit most strongly in the middle of the visible portion of the spectrum, which is why the Sun looks yellow or white in color. A cooler star, such as Betelgeuse (surface temperature 3400 K), looks red because it emits much more red light than blue light. A hotter star, such as Sirius (surface temperature 9400 K), emits a little more blue light than red light and therefore has a slightly blue color to it.

Astronomers can measure surface temperature fairly precisely by comparing a star's apparent brightness in two different colors of light. For example, by comparing the amount of blue light and red light coming from Sirius, astronomers can measure how much more blue light it emits than red light. Because thermal radiation spectra have a very distinctive shape (again, see Figure 5.19), this difference in blue and red light output allows astronomers to calculate surface temperature.

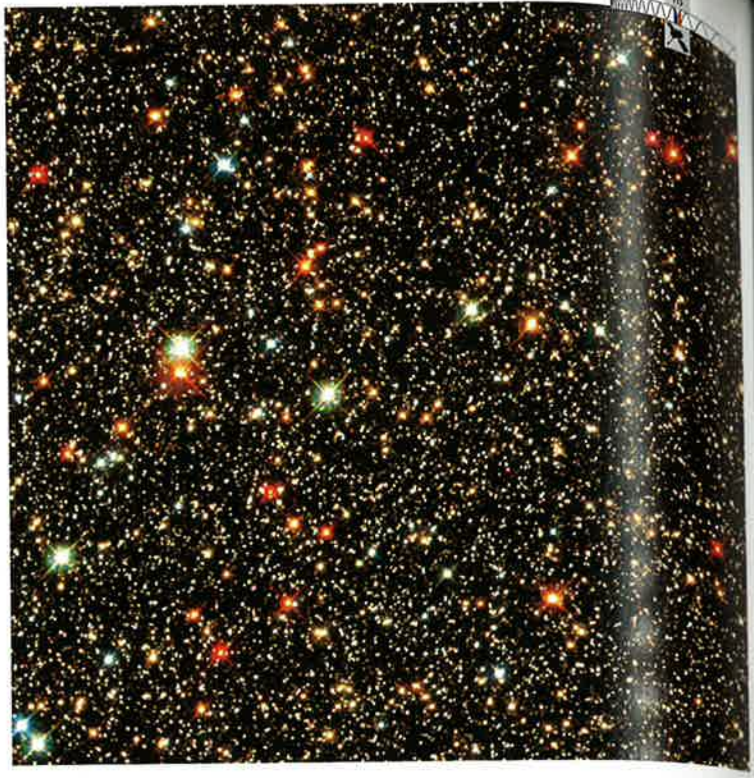


FIGURE 15.4 Interactive Photo This Hubble Space Telescope photo shows a wide variety of stars that differ in color and brightness. Most of the stars in this photo are at roughly the same distance, about 2000 light-years from the center of our galaxy. Clouds of gas and dust obscure our view of visible light from most of our galaxy's central regions, but a gap in the clouds allows us to see the stars in this photo.

Spectral Type and Temperature A star's spectral lines provide a second way to measure its surface temperature. Moreover, because interstellar dust can affect the apparent colors of stars, temperatures determined from spectral lines are generally more accurate than temperatures determined from colors alone. Stars displaying spectral lines of highly ionized elements must be fairly hot, because it takes a high temperature to ionize atoms. Stars displaying spectral lines of molecules must be relatively cool, because molecules break apart into individual atoms unless they are at relatively cool temperatures. The types of spectral lines present in a star's spectrum therefore provide a direct measure of the star's surface temperature.

Astronomers classify stars according to surface temperature by assigning a **spectral type** determined from the spectral lines present in a star's spectrum. The hottest stars, with the bluest colors, are called spectral type O, followed in order of declining surface temperature by spectral types B, A, F, G, K, and M.* The traditional mnemonic for remembering this sequence, OBAFGKM, is "Oh, Be A Fine Girl/Guy, Kiss Me!" Table 15.1 (pp. 500–501) summarizes the characteristics of each spectral type.

*The sequence of spectral types has recently been extended beyond type M to include spectral types L and T, representing starlike objects even cooler than stars of spectral type M. However, as we will see in Chapter 16, most of these objects are not true, hydrogen-burning stars.

COMMON MISCONCEPTIONS

Photos of Stars

Photographs of stars, star clusters, and galaxies convey a great deal of information, but they also contain a few artifacts that are not real. For example, different stars seem to have different sizes in photographs such as Figure 15.4, but stars are so far away that they should all appear as mere points of light. The sizes are an artifact of how our instruments record light. Bright stars tend to be overexposed in photographs, making them appear larger in size than dimmer stars. Overexposure also explains why the centers of globular clusters and galaxies usually look like big blobs in photographs: The central regions of these objects contain many more stars than the outskirts, and the combined light of so many stars tends to get overexposed to make a big blob.

Spikes around bright stars in photographs, often making the pattern of a cross with a star at the center, are another such artifact. You can see these spikes around many of the brightest stars in Figure 15.4. These spikes are not real but rather are created by the interaction of starlight with the supports holding the secondary mirror in the telescope [Section 6.2]. The spikes generally occur only with point sources of light like stars, and not with larger objects like galaxies. When you look at a photograph showing many galaxies (for example, Figure 20.1), you can tell which objects are stars by looking for the spikes.

Each spectral type is subdivided into numbered subcategories (such as B0, B1, ..., B9). The larger the number, the cooler the star. For example, the Sun is designated spectral type G2, which means it is slightly hotter than a G3 star but cooler than a G1 star.

The range of surface temperatures for stars is much narrower than the range of luminosities. The coolest stars, of spectral type M, have surface temperatures as low as 3000 K. The hottest stars, of spectral type O, have surface temperatures that can exceed 40,000 K. Cool, red stars are much more common than hot, blue stars.

THINK ABOUT IT

Invent your own mnemonic for the OBAFGKM sequence. To help get you thinking, here are two examples: (1) Only Bungling Astronomers Forget Generally Known Mnemonics and (2) Only Business Acts For Good, Karl Marx.

History of the Spectral Sequence You may wonder why the spectral types follow the peculiar order of OBAFGKM. The answer lies in the history of stellar spectroscopy.

Astronomical research never paid well, and many astronomers of the 1800s were able to do research only because of family wealth. One such astronomer was Henry Draper (1837–1882), an early pioneer of stellar spectroscopy. After Draper died in 1882, his widow made a series of large donations to Harvard College Observatory for the purpose of building on his work. The observatory director, Edward Pickering (1846–1919), used the gifts to improve the facilities and to hire numerous assistants, whom he called "computers." Pickering added money of his own, as did other wealthy donors.

Most of Pickering's hired computers were women who had studied physics or astronomy at women's colleges such as Wellesley and Radcliffe. Women had few opportunities to advance in science at the time. Harvard, for example, did not allow women to enroll as students and would not hire them as faculty. Pickering's project of studying and classifying stellar spectra provided plenty of work and opportunity for his computers, and many of the Harvard Observatory women ended up among the most prominent astronomers of the late 1800s and early 1900s.

One of the first computers was Williamina Fleming (1857–1911). Following Pickering's suggestion, Fleming classified stellar spectra according to the strength of their hydrogen lines: type A for the strongest hydrogen lines, type B for slightly weaker hydrogen lines, and so on, to type O, for stars with the weakest hydrogen lines. Pickering published Fleming's classifications of more than 10,000 stars in 1890.

As more stellar spectra were obtained and the spectra were studied in greater detail, it became clear that the classification scheme based solely on hydrogen lines was inadequate. Ultimately, the task of finding a better classification scheme fell to Annie Jump Cannon (1863–1941), who joined Pickering's team in 1896 (Figure 15.5). Building on the work of Fleming and another of Pickering's computers, Antonia Maury (1866–1952), Cannon soon realized that the spectral classes fell into a natural order—but not the alphabetical order determined by hydrogen lines alone. Moreover, she found that some of the original classes overlapped others and could be eliminated. Cannon discovered that the natural sequence consisted of just a few of Pickering's original classes in the order OBAFGKM and also added the subdivisions by number.

Cannon became so adept that she could properly classify a stellar spectrum with little more than a momentary glance. In the course of her career, she personally classified more than 400,000 stars. She became the first woman ever awarded

FIGURE 15.5 Women astronomers pose with Edward Pickering at Harvard College Observatory in 1913. Annie Jump Cannon is fifth from the left in the back row.

an honorary degree by Oxford University, and in 1929 the League of Women Voters named her one of the 12 greatest living American women.

The astronomical community adopted Cannon's system of stellar classification in 1910. However, no one at that time knew *why* spectra followed the OBAFGKM sequence. Many astronomers guessed, incorrectly, that the different sets of spectral lines indicated different compositions for the stars. The correct answer—that all stars are made primarily of hydrogen and helium and that a star's surface temperature determines the strength of its spectral lines—was discovered at Harvard Observatory by Cecilia Payne-Gaposchkin (1900–1979).

Cecilia Payne-Gaposchkin

Relying on insights from what was then the newly developing science of quantum mechanics, Payne-Gaposchkin showed that the differences in spectral lines from star to star merely reflected changes in the ionization level of the emitting atoms. For example, O stars have weak hydrogen lines because, at their high surface temperatures, nearly all their hydrogen is ionized. Without an electron to "jump" between energy levels, ionized hydrogen can neither emit nor absorb its usual specific wavelengths of light. At the other end of the spectral sequence, M stars are cool enough for some particularly stable molecules to form, explaining their strong molecular absorption lines. Payne-Gaposchkin described her work and her conclusions in a dissertation published in 1925 and that was later called "undoubtedly the most brilliant Ph.D. thesis ever written in astronomy."

TABLE 15.1 The Spectral Sequence

Spectral Type	Example(s)	Temperature Range
O	Stars of Orion's Belt	>30,000 K
B	Rigel	30,000 K–10,000 K
A	Sirius	10,000 K–7500 K
F	Polaris	7500 K–6000 K
G	Sun, Alpha Centauri A	6000 K–5000 K
K	Arcturus	5000 K–3500 K
M	Betelgeuse, Proxima Centauri	<3500 K

How do we measure stellar masses?

Mass is generally more difficult to measure than surface temperature or luminosity. The most dependable method for "weighing" a star relies on Newton's version of Kepler's third law [Section 4.4]. Recall that this law can be applied only when we can observe one object orbiting another, and it requires that we measure both the orbital period and the average orbital distance of the orbiting object. For stars, these requirements generally mean that we can apply the law to measure masses only in **binary star systems**—systems in which two stars continually orbit one another. Before we consider how we determine the orbital periods and distances needed to use Newton's version of Kepler's third law, let's look briefly at the different types of binary star systems that we can observe.

Types of Binary Star Systems Surveys show that about half of all stars orbit a companion star of some kind and are therefore members of binary star systems. These star systems fall into three classes:

- A **visual binary** is a pair of stars that we can see distinctly (with a telescope) as the stars orbit each other. Sometimes we observe a star slowly shifting position in the sky as if it were a member of a visual binary, but its companion is too dim to be seen. For example, slow shifts in the position of Sirius, the brightest star in the sky, revealed it to be a binary star long before its companion was discovered (Figure 15.6).

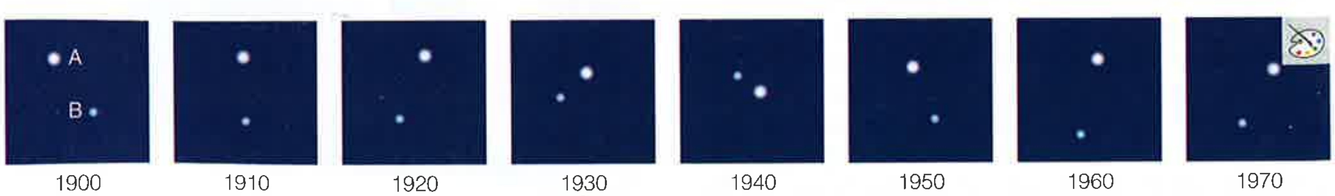
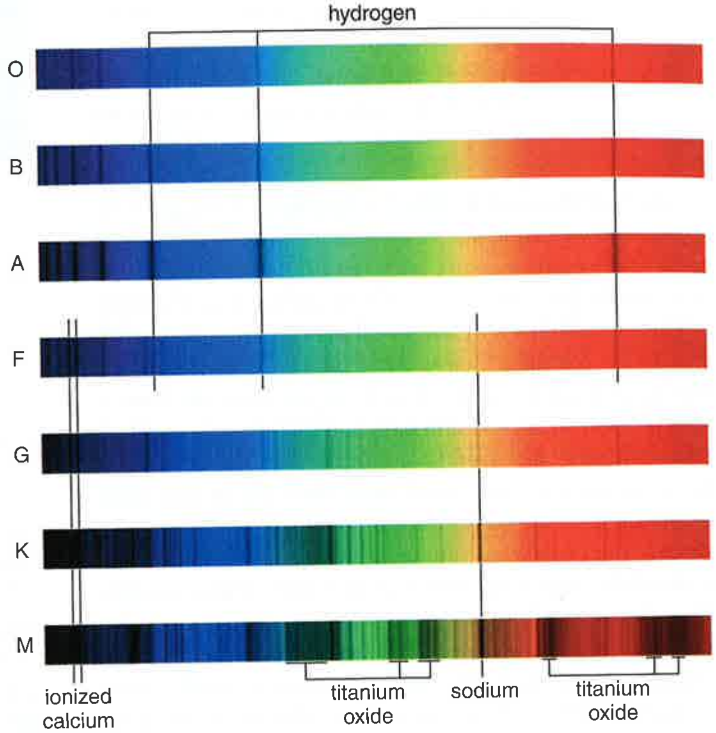


FIGURE 15.6 Each frame represents the relative positions of Sirius A and Sirius B at 10-year intervals from 1900 to 1970. The back-and-forth "wobble" of Sirius A allowed astronomers to infer the existence of Sirius B even before the two stars could be resolved in telescopic photos. The average orbital separation of the binary system is about 20 AU.

Key Absorption Line Features	Brightest Wavelength (Color)	Typical Spectrum
Lines of ionized helium, weak hydrogen lines	<97 nm (ultraviolet)*	O
Lines of neutral helium, moderate hydrogen lines	97–290 nm (ultraviolet)*	B
Very strong hydrogen lines	290–390 nm (violet)*	A
Moderate hydrogen lines, moderate lines of ionized calcium	390–480 nm (blue)*	F
Weak hydrogen lines, strong lines of ionized calcium	480–580 nm (yellow)	G
Lines of neutral and singly ionized metals, some molecules	580–830 nm (red)	K
Strong molecular lines	>830 nm (infrared)	M



*All stars above 6000 K look more or less white to the human eye because they emit plenty of radiation at all visible wavelengths.

- An **eclipsing binary** is a pair of stars that orbit in the plane of our line of sight (Figure 15.7). When neither star is eclipsed, we see the combined light of both stars. When one star eclipses the other, the apparent brightness of the system drops because some of the light is blocked from our view. A *light curve,* or graph of apparent brightness against time, reveals the pattern of the eclipses. The most famous example of an eclipsing binary is Algol, the "demon star" in the constellation Perseus (*algol* is Arabic for "the ghoul"). Algol's brightness drops to only a third of its usual level for a few hours about every 3 days as the brighter of its two stars is eclipsed by its dimmer companion.

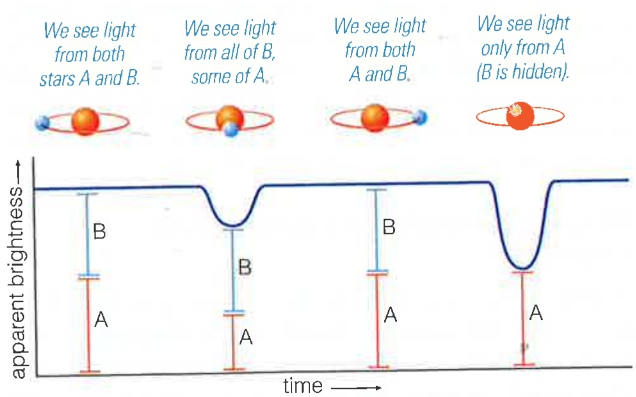


FIGURE 15.7 Interactive Figure The apparent brightness of an eclipsing binary system drops when either star eclipses the other.

- If a binary system is neither visual nor eclipsing, we may be able to detect its binary nature by observing Doppler shifts in its spectral lines [Section 5.5]. Such systems are called **spectroscopic binary** systems. If one star is orbiting another, it periodically moves toward us and away from us in its orbit. Its spectral lines show blueshifts and redshifts as a result of this motion (Figure 15.8). Sometimes we see two sets of lines shifting back and forth—one set from each of the two stars in the system (a *double-lined* spectroscopic binary). Other times we see a set of shifting lines

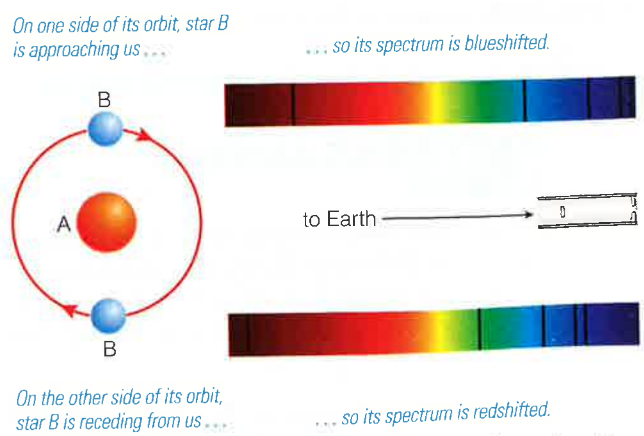


FIGURE 15.8 Interactive Figure The spectral lines of a star in a binary system are alternately blueshifted as it comes toward us in its orbit and redshifted as it moves away from us.

from only one star because its companion is too dim to be detected (a *single-lined* spectroscopic binary).

Some star systems combine two or more of these binary types. For example, telescopic observations reveal Mizar (the second star in the handle of the Big Dipper) to be a visual binary. Spectroscopy then shows that each of the two stars in the visual binary is itself a spectroscopic binary (Figure 15.9).

Measuring Masses in Binary Systems Even for a binary system, we can apply Newton's version of Kepler's third law only if we can measure both the orbital period and the separation of the two stars. Measuring orbital period is fairly easy. In a visual binary, we simply observe how long each orbit takes. In an eclipsing binary, we measure the time between eclipses. In a spectroscopic binary, we measure the time it takes the spectral lines to shift back and forth.

Determining the average separation of the stars in a binary system is usually much more difficult. In rare cases we can measure the separation directly; otherwise, we can calculate the separation only if we know the actual orbital speeds of the stars from their Doppler shifts. Unfortunately, a Doppler shift tells us only the portion of a star's velocity that is directed toward us or away from us (see Figure 5.24). Because orbiting stars generally do not move directly along our line of sight, their actual velocities can be significantly greater than those we measure through the Doppler effect.

The exceptions are eclipsing binary stars. Because these stars orbit in the plane of our line of sight, their Doppler shifts can tell us their true orbital velocities.* Eclipsing binaries are therefore particularly important to the study of stellar masses. As an added bonus, eclipsing binaries allow us to measure stellar radii directly. Because we know how fast the stars are moving across our line of sight as one eclipses the other, we can determine their radii by timing how long each eclipse lasts.

Through careful observations of eclipsing binaries and other binary star systems, astronomers have established the masses of many different kinds of stars. The overall range extends from as little as 0.08 times the mass of the Sun ($0.08M_{Sun}$) to about 150 times the mass of the Sun ($150M_{Sun}$). We'll discuss the reasons for that mass range in Chapter 16.

MA The Hertzsprung-Russell Diagram Tutorial, Lessons 1–3

15.2 PATTERNS AMONG STARS

We have seen that stars come in a wide range of luminosities, surface temperatures, and masses. But are these characteristics randomly distributed among stars, or can we find patterns that might tell us something about stellar lives? The key that finally unlocked the secrets of stars was the development of an appropriate classification system.

Before reading any further, take another look at Figure 15.4 and think about how you would classify these stars. Almost all of them are at nearly the same distance from Earth, so we can compare their true luminosities by looking at their apparent brightnesses in the photograph. If you look closely, you might notice a couple of important patterns:

- Most of the very brightest stars are reddish in color.
- If you ignore those relatively few bright red stars, there's a general trend to the luminosities and colors among all the rest of the stars: The brighter ones are white with a little bit of blue tint, the more modest ones are similar to our Sun in color with a yellowish white tint, and the dimmest ones are barely visible specks of red.

Keeping in mind that colors tell us about surface temperature—blue is hotter and red is cooler—you can see that these patterns must be telling us about relationships between surface temperature and luminosity.

Danish astronomer Ejnar Hertzsprung and American astronomer Henry Norris Russell discovered these relationships in the first decade of the 20th century. Building upon the work of Annie Jump Cannon and others, Hertzsprung and Russell independently decided to make graphs of stellar properties by plotting stellar luminosities on one axis and spectral types on the other. These graphs revealed previously unsuspected patterns among the properties of stars and ultimately unlocked the secrets of stellar life cycles.

What is a Hertzsprung-Russell diagram?

Graphs of the type made by Hertzsprung and Russell are now called **Hertzsprung-Russell (H-R) diagrams**. These diagrams quickly became one of the most important tools in astronomical research, and they remain central to the study of stars today.

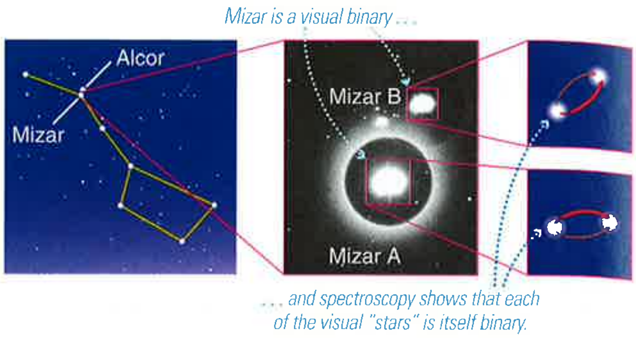


FIGURE 15.9 Mizar looks like one star to the naked eye but is actually a system of four stars. Through a telescope, Mizar appears to be a visual binary made up of two stars, Mizar A and Mizar B, that gradually change positions, indicating that they orbit each other every few thousand years. Moreover, each of these two "stars" is itself a spectroscopic binary, making a total of four stars. (The star Alcor appears to be very close to Mizar to the naked eye but does *not* orbit it. The ring around Mizar A is an artifact of the photographic process, not a real feature.)

*In other binaries, we can calculate an actual orbital velocity from the velocity obtained by the Doppler effect if we also know the system's orbital inclination. Astronomers have developed techniques for determining orbital inclination in a relatively small number of cases.

Basics of the H-R Diagram Figure 15.10 displays an example of an H-R diagram. All you need to know to plot a star on an H-R diagram is its luminosity and its spectral type.

- The horizontal axis represents stellar surface temperature, which, as we've discussed, corresponds to spectral type. Temperature *decreases* from left to right because Hertzsprung and Russell based their diagrams on the spectral sequence OBAFGKM.
- The vertical axis represents stellar luminosity, in units of the Sun's luminosity (L_{Sun}). Stellar luminosities span a wide range, so we keep the graph compact by making each tick mark represent a luminosity 10 times as large as the prior tick mark.

Each location on the diagram represents a unique combination of spectral type and luminosity. For example, the dot representing the Sun in Figure 15.10 corresponds to the Sun's spectral type, G2, and its luminosity, $1L_{Sun}$. Because luminosity increases upward on the diagram and surface temperature increases leftward, stars near the upper left are hot and luminous. Similarly, stars near the upper right are cool and luminous, stars near the lower right are cool and dim, and stars near the lower left are hot and dim.

THINK ABOUT IT

Explain how the colors of the stars in Figure 15.10 help indicate stellar surface temperature. Do these colors tell us anything about interior temperatures? Why or why not?

The H-R diagram also provides direct information about stellar radii, because a star's luminosity depends on both its surface temperature and its surface area or radius (see Mathematical Insight 15.5). If two stars have the same surface temperature, one can be more luminous than the other only if it is larger in size. Stellar radii therefore must increase as we go

MATHEMATICAL INSIGHT 15.4

Measuring Stellar Masses

We can apply Newton's version of Kepler's third law (see Mathematical Insight 4.3) to measure the masses of stars in binary systems if we know the orbital period p and semimajor axis a of the binary system. The orbital period is generally easy to measure, and we can often calculate a from Doppler-shift measurements of the stars' orbital velocities. For a binary system in which one star traces a circle of radius a around its companion, meaning that it travels a distance $2\pi a$ in one orbital period p, the star's orbital velocity relative to its companion is

$$v = \frac{\text{distance traveled in one orbit}}{\text{period of one orbit}} = \frac{2\pi a}{p}$$

Solving for a, we find

$$a = \frac{pv}{2\pi}$$

Once we know both p and a, Newton's version of Kepler's third law allows us to calculate the *sum* of the masses of the two stars ($M_1 + M_2$). We can calculate individual masses by comparing the orbital velocities of the two stars around the system's center of mass.

EXAMPLE: The spectral lines of two stars in an eclipsing binary system with a circular orbit shift back and forth with a period of 2 years ($p = 6.3 \times 10^7$ seconds). The lines of one star (Star 1) shift twice as far as the lines of the other (Star 2). The Doppler shift indicates an orbital speed of $v = 100{,}000$ m/s for Star 1 relative to Star 2. What are the masses of the two stars?

SOLUTION:

Step 1 Understand: We can find the sum of the masses with Newton's version of Kepler's third law, which reads

$$p^2 = \frac{4\pi^2}{G(M_1 + M_2)} a^3$$

We rearrange this equation to find the masses:

$$M_1 + M_2 = \frac{4\pi^2}{G} \times \frac{a^3}{p^2}$$

To use this law, we need to know the star system's orbital period p and semimajor axis a. We are given the system's orbital period p, and because the orbit is circular, we can find the semimajor axis a as described above from the velocity v of Star 1 relative to Star 2. Once we use this information to calculate the sum of the masses ($M_1 + M_2$), we can determine the relative masses of the stars in the system by comparing their Doppler shifts: Because the lines of Star 1 shift twice as far as those of Star 2, we know that Star 1 moves twice as fast as Star 2, and hence that Star 1 is half as massive as Star 2.

Step 2 Solve: First, we find the semimajor axis a of the system from the system's orbital velocity v:

$$a = \frac{pv}{2\pi} = \frac{(6.3 \times 10^7\ \text{s}) \times (100{,}000\ \text{m/s})}{2\pi}$$
$$= 1.0 \times 10^{12}\ \text{m}$$

Second, we use this value, the value of the gravitational constant G (see Appendix A), and the given orbital period ($p = 6.3 \times 10^7$ s) to find the sum of the masses with Newton's version of Kepler's third law:

$$M_1 + M_2 = \frac{4\pi^2}{\left(6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \times \text{s}^2}\right)} \times \frac{(1.0 \times 10^{12}\ \text{m})^3}{(6.3 \times 10^7\ \text{s})^2}$$
$$= 1.5 \times 10^{32}\ \text{kg}$$

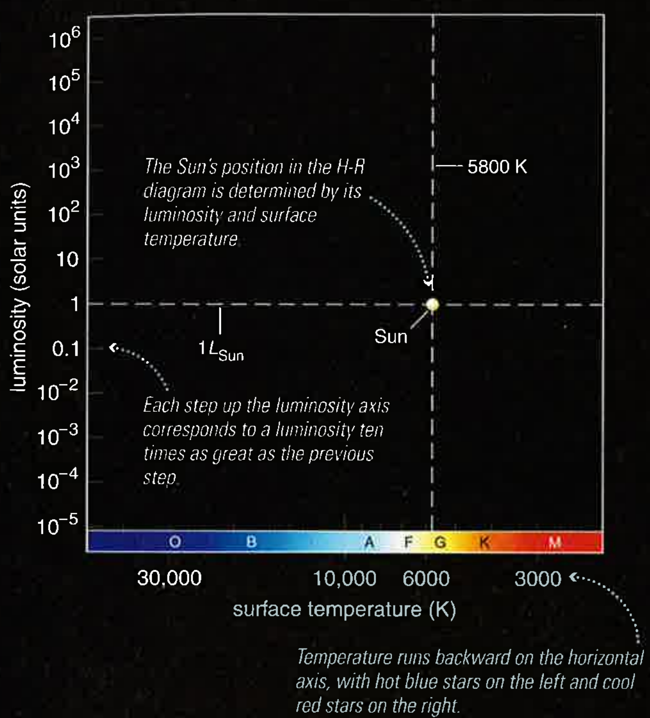
We have found that the two stars have a combined mass of 1.5×10^{32} kg; we know from the Doppler shifts that Star 2 is twice as massive as Star 1, which means that Star 2 has a mass of 1.0×10^{32} kg and Star 1 has a mass of 0.5×10^{32} kg.

Step 3 Explain: The masses will be more meaningful if we convert them from kilograms to solar masses, which we do by dividing by the Sun's mass of 2×10^{30} kg. Doing so, we find that this binary system consists of one star of mass $50M_{Sun}$ and another star of mass $25M_{Sun}$.

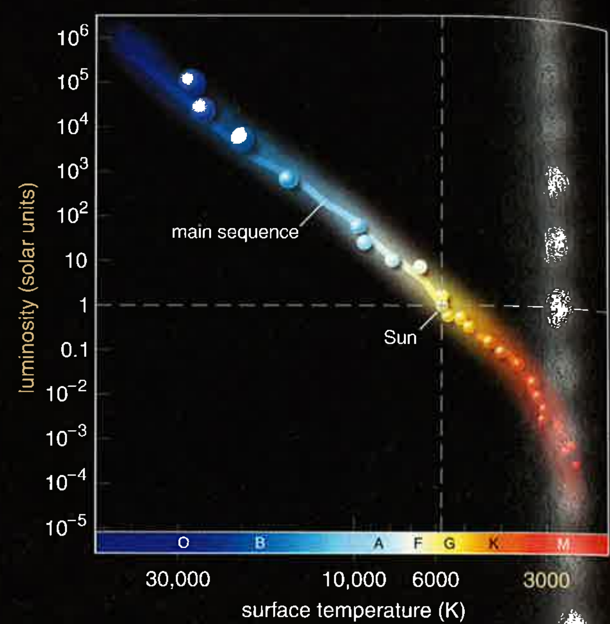
COSMIC CONTEXT FIGURE 15.10 *Reading an H-R Diagram*

Hertzsprung-Russell (H-R) diagrams are very important tools in astronomy because they reveal key relationships among the properties of stars. An H-R diagram is made by plotting stars according to their surface temperatures and luminosities. This figure shows a step-by-step approach to building an H-R diagram.

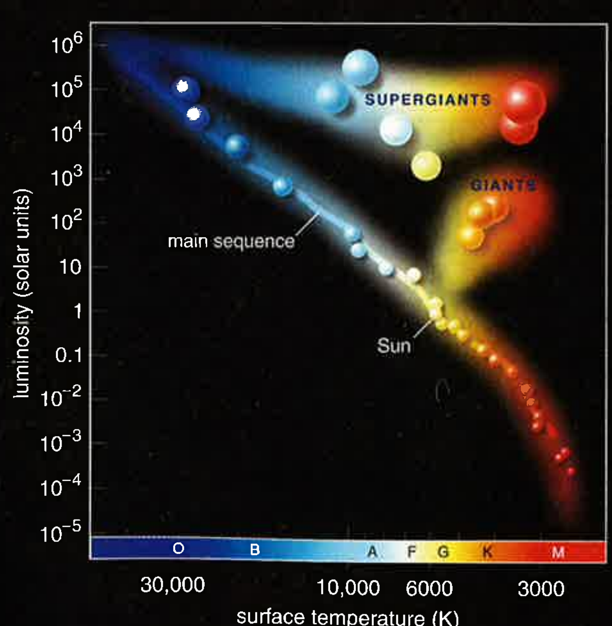
① **An H-R Diagram Is a Graph:** A star's position along the horizontal axis indicates its surface temperature, which is closely related to its color and spectral type. Its position along the vertical axis indicates its luminosity.



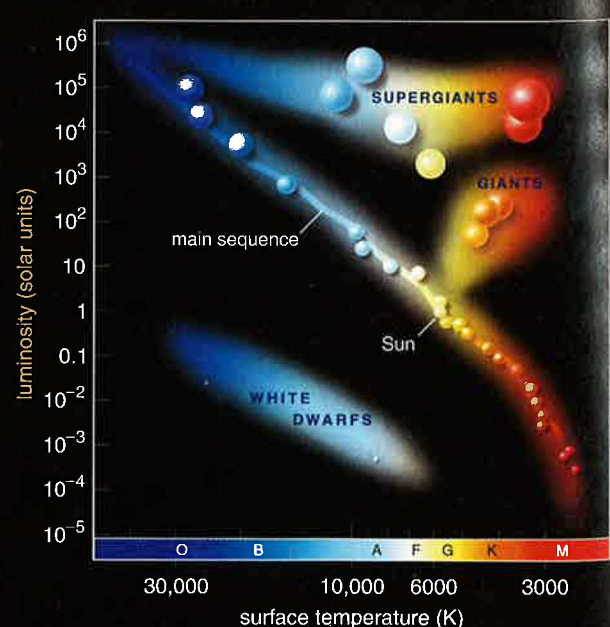
② **Main Sequence:** Our Sun falls along the main sequence, a line of stars extending from the upper left of the diagram to the lower right. Most stars are main-sequence stars, which shine by fusing hydrogen into helium in their cores.



③ **Giants and Supergiants:** Stars in the upper right of an H-R diagram are more luminous than main-sequence stars of the same surface temperature. They must therefore be very large in radius, which is why they are known as *giants* and *supergiants*.

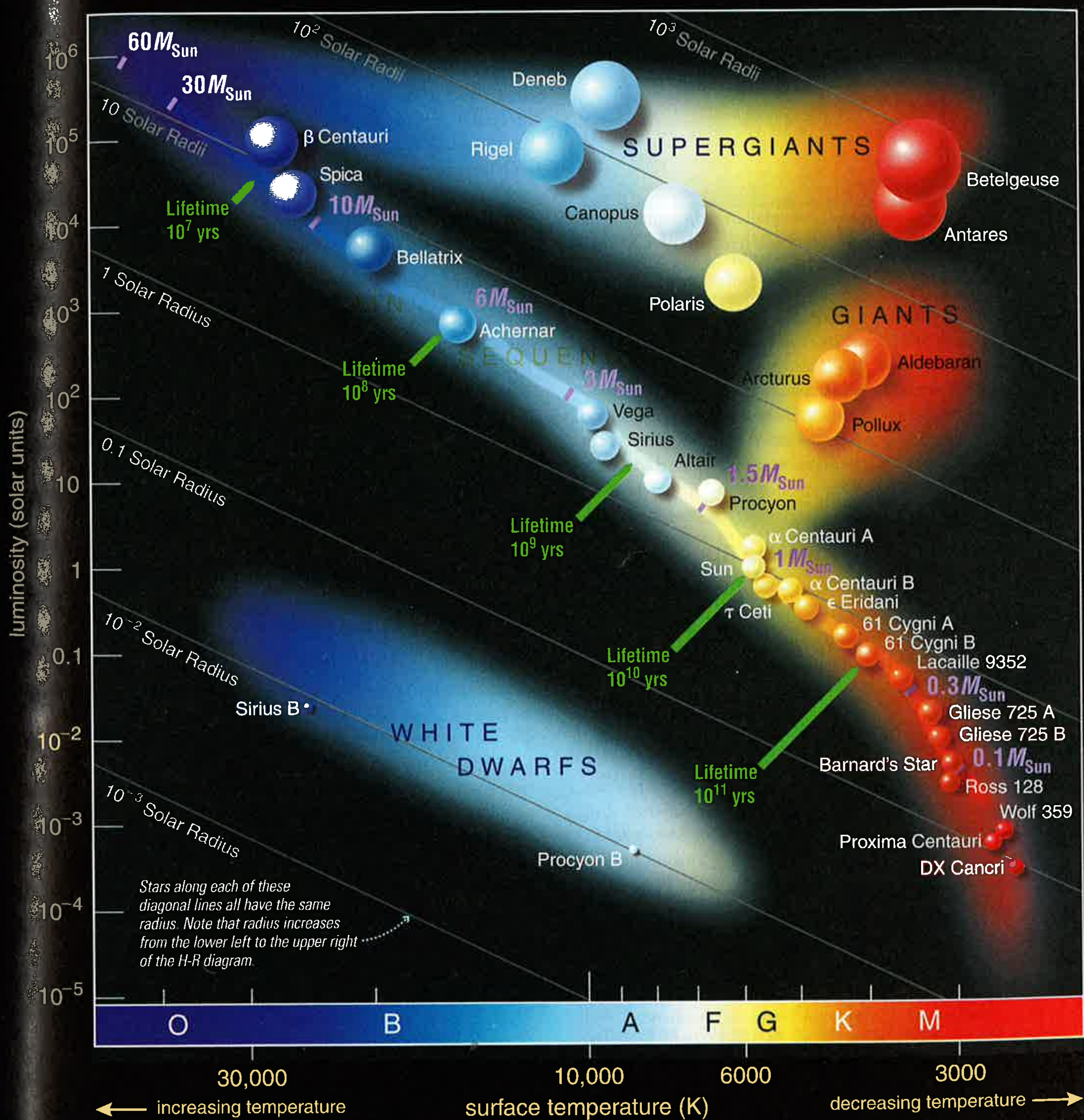


④ **White Dwarfs:** Stars in the lower left have high surface temperatures, dim luminosities, and small radii. These stars are known as *white dwarfs*.



(5) **Masses on the Main Sequence:** Stellar masses (purple labels) decrease from the upper left to the lower right on the main sequence.

(6) **Lifetimes on the Main Sequence:** Stellar lifetimes (green labels) increase from the upper left to lower right on the main sequence: High-mass stars live shorter lives because their high luminosities mean they burn through their nuclear fuel more quickly.



from the high-temperature, low-luminosity corner on the lower left of the H-R diagram to the low-temperature, high-luminosity corner on the upper right. Notice the diagonal lines that represent different stellar radii in Figure 15.10.

Patterns in the H-R Diagram Stars do not fall randomly throughout an H-R diagram like Figure 15.10 but instead cluster into four major groups:

- Most stars fall somewhere along the **main sequence**, the prominent streak running from the upper left to the lower right on the H-R diagram. Notice that our Sun is one of these *main-sequence stars*.
- The stars in the upper right are called **supergiants** because they are very large in addition to being very bright.
- Just below the supergiants are the **giants**, which are somewhat smaller in radius and lower in luminosity (but still much larger and brighter than main-sequence stars of the same spectral type).
- The stars near the lower left are small in radius and appear white in color because of their high temperatures. We call these stars **white dwarfs**.

Luminosity Classes In addition to the four major groups we've just listed, stars sometimes fall into "in-between" categories. For more precise work, astronomers therefore assign each star to a **luminosity class**, designated with a Roman numeral from I to V. The luminosity class describes the region of the H-R diagram in which the star falls; thus, despite the name, a star's luminosity class is more closely related to its size than to its luminosity. The basic luminosity classes are I for supergiants, III for giants, and V for main-sequence stars. Luminosity classes II and IV are intermediate to the others. For example, luminosity class IV represents stars with radii larger than those of main-sequence stars but not quite large enough to qualify them as giants. Table 15.2 summarizes the luminosity classes. White dwarfs fall outside this classification system and instead are often assigned the luminosity class "wd."

TABLE 15.2 Stellar Luminosity Classes

Class	Description
I	Supergiants
II	Bright giants
III	Giants
IV	Subgiants
V	Main-sequence stars

Complete Stellar Classification We have now described two different ways of categorizing stars:

- A star's *spectral type*, designated by one of the letters OBAFGKM, tells us its surface temperature and color. O stars are the hottest and bluest, while M stars are the coolest and reddest.
- A star's *luminosity class*, designated by a Roman numeral, is based on its luminosity but also tells us about the star's radius. Luminosity class I stars have the largest radii, with radii decreasing to luminosity class V.

MATHEMATICAL INSIGHT 15.5

Calculating Stellar Radii

Although we can rarely measure stellar radii directly, we can calculate radii using the laws of thermal radiation. As given in Mathematical Insight 5.2, the amount (power) of thermal radiation emitted by a star of surface temperature T (on the Kelvin scale) is

$$\text{emitted power (per square meter of surface)} = \sigma T^4$$

where the constant $\sigma = 5.7 \times 10^{-8}$ watt/($m^2 \times K^4$).

The luminosity L of a star is its power per unit area multiplied by its total surface area, and a star of radius r has surface area $4\pi r^2$. That is,

$$L = 4\pi r^2 \times \sigma T^4$$

With a bit of algebra, we can solve this formula for the star's radius r:

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

EXAMPLE: The red supergiant star Betelgeuse has a luminosity of $38{,}000 L_{Sun}$ and a surface temperature of about 3400 K. What is its radius?

SOLUTION:

Step 1 Understand: We are given Betelgeuse's luminosity L and surface temperature T, so we can use the above formula to find its radius as long as we make the units consistent; looking at the units of the constant σ, we see that we will need to convert the luminosity to watts.

Step 2 Solve: First, we convert the given luminosity into watts. Remembering that $L_{Sun} = 3.8 \times 10^{26}$ watts, we find

$$L_{Bet} = 38{,}000 \times L_{Sun} = 38{,}000 \times 3.8 \times 10^{26} \text{ watts} = 1.4 \times 10^{31} \text{ watts}$$

Now we can use our formula to calculate radius:

$$r = \sqrt{\frac{L}{4\pi\sigma T^4}} = \sqrt{\frac{1.4 \times 10^{31} \text{ watts}}{4\pi \times \left(5.7 \times 10^{-8} \frac{\text{watt}}{m^2 \times K^4}\right) \times (3400 \text{ K})^4}} = \sqrt{\frac{1.4 \times 10^{31} \text{ watts}}{9.6 \times 10^{7} \frac{\text{watts}}{m^2}}} = 3.8 \times 10^{11} \text{ m}$$

Step 3 Explain: Betelgeuse has a radius of about 380 billion meters, which is 380 million kilometers. We can make this number more meaningful by comparing it to the Earth-Sun distance of about 150 million kilometers (1 AU); notice that Betelgeuse has a radius more than twice the Earth-Sun distance, which means the orbits of all the inner planets of our solar system could fit easily inside Betelgeuse.

We use both spectral type and luminosity class to fully classify a star. For example, the complete classification of our Sun is G2 V. The G2 spectral type means it is yellow-white in color, and the luminosity class V means it is a hydrogen-burning, main-sequence star. Betelgeuse is M2 I, making it a red supergiant. Proxima Centauri is M5 V—similar in color and surface temperature to Betelgeuse, but far dimmer because of its much smaller size.

THINK ABOUT IT

By studying Figure 15.10, determine the approximate spectral type, luminosity class, and radius of the following stars: Bellatrix, Vega, Antares, Pollux, and Proxima Centauri.

What is the significance of the main sequence?

Most of the stars we observe, including our Sun, have properties that place them on the main sequence of the H-R diagram. You can see in Figure 15.10 that high-luminosity main-sequence stars have hot surfaces and low-luminosity main-sequence stars have cooler surfaces. The significance of this relationship between luminosity and surface temperature became clear when astronomers started measuring the masses of main-sequence stars in binary star systems. These measurements showed that a star's position along the main sequence is closely related to its mass. We now know that all stars along the main sequence are fusing hydrogen into helium in their cores, just like the Sun, and that a main-sequence star's mass determines its other properties because it sets the balancing point at which energy produced by fusion in the core equals the output of radiative energy from the star's surface, allowing gravitational equilibrium to remain steady.

Masses Along the Main Sequence If you look along the main sequence in Figure 15.10, you'll notice purple labels indicating stellar masses and green labels indicating stellar lifetimes. To make them easier to see, Figure 15.11 repeats the same data but shows only the main sequence rather than the entire H-R diagram.

Let's focus first on mass: Notice that *stellar masses decrease downward along the main sequence.* At the upper end of the main sequence, the hot, luminous O stars can have masses as high as 150 times that of the Sun ($150M_{Sun}$). On the lower end, cool, dim M stars may have as little as 0.08 times the mass of the Sun ($0.08M_{Sun}$). Many more stars fall on the lower end of the main sequence than on the upper end, which tells us that low-mass stars are much more common than high-mass stars.

The orderly arrangement of stellar masses along the main sequence tells us that *mass* is the most important attribute of a hydrogen-burning star. As we've discussed, mass is crucially important because it sets the fusion rate at which pressure and gravity can remain in balance. The nuclear fusion rate, and hence the luminosity, is very sensitive to mass. For example, a $10M_{Sun}$ star on the main sequence is about 10,000 times as luminous as the Sun.

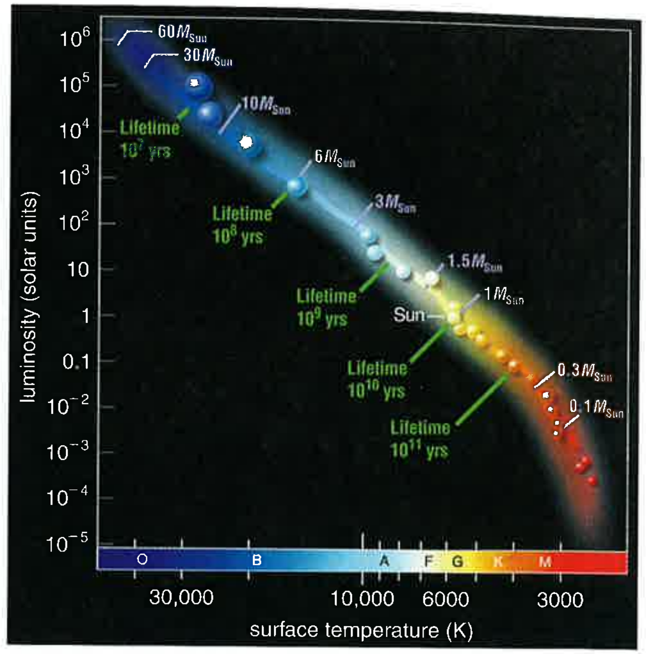


FIGURE 15.11 The main sequence from Figure 15.10 is isolated here so that you can more easily see how masses and lifetimes vary along it. Notice that more massive hydrogen-burning stars are brighter and hotter but have shorter lifetimes. (Stellar masses are given in units of solar masses: $1M_{Sun} = 2 \times 10^{30}$ kg.)

The relationship between mass and surface temperature is a little more subtle. In general, a very luminous star must be extremely large or have an unusually high surface temperature, or some combination of both. The most massive main-sequence stars are many thousands of times more luminous than the Sun but only about 10 times the size of the Sun in radius. Their surfaces must be significantly hotter than the Sun's surface to account for their high luminosities. Main-sequence stars more massive than the Sun therefore have higher surface temperatures than the Sun, and those less massive than the Sun have lower surface temperatures. That is why the main sequence slices diagonally from the upper left to the lower right on the H-R diagram.

The fact that mass, surface temperature, and luminosity are all related means that we can estimate a main-sequence star's mass just by knowing its spectral type. For example, any hydrogen-burning, main-sequence star that has the same spectral type as the Sun (G2) must have about the same mass and luminosity as the Sun. Similarly, any main-sequence star of spectral type B1 must have about the same mass and luminosity as Spica (see Figure 15.10). Note that only main-sequence stars follow this simple relationship between mass, temperature, and luminosity; it does not hold for giants, supergiants, or white dwarfs.

Lifetimes Along the Main Sequence A star is born with a limited supply of core hydrogen and therefore can remain as a hydrogen-fusing, main-sequence star for only a limited time—the star's **main-sequence lifetime.** Because stars spend the vast majority of their lives as main-sequence stars,